On the Difference Between Updating a Knowledge Base and Revising it

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- Intuitively, R5 and R6 say that revision should be accomplished with minimal change.

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3. $\psi \leftrightarrow \phi$ implies $\leq_\psi = \leq_\phi$.

Semantic Characterization of Revision

- Let $\mathcal{M} \subseteq \mathcal{I}$. I is **minimal in** $\mathcal{M}$ **with respect to** $\leq_\psi$ if $I \in \mathcal{M}$ and there does not exist $I' \in \mathcal{M}$, such that $I' <_\psi I$.

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Theorem

A revision operator $\circ$ satisfies Axioms R1-R6 iff there exists a faithful assignment that maps each KB ψ to a total pre-order $\leq_\psi$, such that $\mathrm{Mod}(\psi \circ \mu) = \mathrm{Min}(\mathrm{Mod}(\mu), \leq_\psi)$.

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- It cannot be repaired by update since a change of worlds when there is no available world to start with leaves us with no worlds!

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- Update operators do not satisfy R2: Result not applicable.

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    the total pre-order analog of Update Theorem may be proven.
- Update is "local", a different ordering is induced by each model of ψ, whereas revision is "global", only one ordering is induced by the whole of ψ.

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  - Postcondition ≡ New knowledge
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  - The effect on KB ψ of performing action (α, β) is

$$\begin{cases} \psi, & \text{if } \psi \not\to \alpha \\ \psi \diamond \beta, & \text{otherwise} \end{cases}$$

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  - the truth of the precondition cannot be determined.
- Maybe (?) as follows:

$$\begin{cases} (\psi \circ \alpha) \diamond \beta, & \text{if } \psi \rightarrow \alpha \\ \psi \circ \neg\alpha, & \text{if } \psi \rightarrow \neg\alpha \\ \psi \bullet \alpha, & \text{if } \psi \not\rightarrow \alpha \text{ and } \psi \not\rightarrow \neg\alpha \end{cases}$$

- $\bullet$ is **contraction** to be discussed next.

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E1 ψ implies $\psi \blacklozenge \mu$;

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E4 If $\psi_1 \leftrightarrow \psi_2$ and $\mu_1 \leftrightarrow \mu_2$, then $\psi_1 \blacklozenge \mu_1 \leftrightarrow \psi_2 \blacklozenge \mu_2$; $(\psi \blacklozenge \mu) \wedge \mu$ implies ψ;

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- Erasure is to contraction as update is to revision.
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- Erasure needs the disjunctive rule E8, but contraction does not.

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- Intuitively, b is already questionable by ψ, so contracting b does not change ψ.

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- Then $\psi \blacklozenge b \leftrightarrow (b \wedge \neg m) \vee \neg b$.
- Intuitively, ψ represents $M_1 = \{b, \neg m\}, M_2 = \{\neg b, m\}$. When b is erased, M_1 changes to M_1 and $M_3 = \{\neg b, \neg m\}$. On the other hand, M_2 stays fixed. Thus, we end up with $\{M_1, M_3, M_2\}$ which is described by $\psi \blacklozenge b$.

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- **Symmetric Erasure:** Suppose the state has changed so that the location of the book is unpredictable. Symmetric Erasure allows us to update the knowledge base accordingly.

Erasure and Update (By analogy with Revision and Contraction)

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- Symmetric Contraction can be defined similarly.

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- What happens if $t' < t$ is left as a problem for future research.