

# Two Learning Schemes in Information Retrieval\*

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Two methods are given to improve weighting schemes by using relevance information of a set of queries. The first method is to estimate parameter values of two independence models in information retrieval --- the binary independence model and the non-binary independence model. The parameters estimated here are used to calculate optimal weights for terms in a different set of queries. Performance of this estimation is compared to the inverse document frequency method, the cosine measure, and the statistical similarity measure. The second method is to learn optimal weights of the non-binary independence model adaptively by a learning formula. Experiments are performed on three different document collections CISI, MEDLARS, and CRN4NUL for both methods, and results are reported. Both methods show improvements compared to the existing weighting schemes. Experimental results show that the second method gives slightly better performance than the first one, and has simpler implementation.

## 1. Introduction

In information retrieval systems, it is common to compute a similarity between a query and a document for retrieval of documents. Many methods of calculating similarities have been presented. Two well-known methods are the inverse document frequency method [Spar] and the cosine measure [SaMc]. Both methods are simple and intuitive. The inverse document frequency method assigns a higher weight to a term which occurs in few documents than a term in many documents. The similarity measure between a document and a query is simply the sum of the weights of the query terms in the document. The cosine measure is the cosine of the angle between the query and the document when they are represented as vectors. In spite of their intuitive appeal, they can not ensure optimality in retrieval.

The binary independence model [YuSa][RoSp] and the non-binary independence model [YuLe] are theoretical models which have been proposed to guarantee optimal retrieval. Optimality here means that a document with higher probability of relevance to a query is assigned a higher similarity than a document with lower probability of relevance. These two models are theoretical because they suppose a priori knowledge about some parameters used in calculation of similarities.

In this paper, the parameter values of these two independence models are estimated from document and term frequencies of terms. The estimation method of the parameter values is discussed and the performances of the estimated independence models are compared to that of the inverse document frequency method, the cosine measure, and the method which has been recently reported [WoYa] and called the statistical similarity measure. In section 2, outlines of two independence models are given. In section 3, we discuss the estimation method of the parameters, and introduce some constraints which should be satisfied by the estimated parameter values and the weighting functions derived from the parameters. The estimated independence models are applied to three different document collections and the performances are shown in section

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4. In section 5, motivations of a learning scheme are described. In section 6, the learning scheme is given and its properties are discussed. In section 7, experimental results of the second method are given. The conclusions of the two methods are given in section 8.

## 2. Two Independence Models

### 2.1. Binary Independence Model

The binary independence model [RoSp][YuSa] ensures the optimality of retrieval described below.

Let  $p(R|D_i)$  be the probability that a document  $D_i$  is relevant to the query and  $f(Q, D_i)$  be the similarity between a query  $Q$  and a document  $D_i$ , where "R" in  $p(R|D_i)$  stands for "relevance". Then the following condition is satisfied by the binary independence model.

$$f(Q, D_i) > f(Q, D_j) \leftrightarrow p(R|D_i) > p(R|D_j)$$

The binary independence model makes the following two assumptions.

- (1) Binary assumption : All document vectors are binary vectors. In other words, each element  $d_{ij}$  is 0 or 1. 0 means the absence of the term  $T_j$  in document  $D_i$  and 1 means the presence of the term  $T_j$  in document  $D_i$ .
- (2) Independence assumption : All terms are statistically independent within the relevant document set and within the irrelevant document set.

Based on the above two assumptions, the weight assigned to term  $T_j$  is given by [RoSp]

$$w_1(T_j) = \log \frac{p_j}{1-p_j} - \log \frac{q_j}{1-q_j} \quad (2.1)$$

where  $p_j$  is the probability that a relevant document has the term  $T_j$  and  $q_j$  is the probability that an irrelevant document has the term  $T_j$ . For a query  $Q$  having the following terms,

$$Q = (T_{k1}, T_{k2}, \dots, T_{kl})$$

and for a document  $D_i = (d_{i1}, d_{i2}, \dots, d_{iM})$  the similarity between the two vectors is

$$\text{sim}(Q, D_i) = \sum_{n=1}^l d_{kn} \cdot w_1(T_{kn}),$$

where  $d_{kn}$  is 0 or 1 because of the binary assumption.

In deriving the weighting function (2.1), we suppose a priori knowledge of  $p_j$  and  $q_j$ . These probability values are unknown in advance under ordinary circumstances. Our aim is to estimate  $p_j$  and  $q_j$  from some other parameters which are easily obtainable.

### 2.2. Non-Binary Independence Model

In the non-binary independence model, the independence assumption in the previous section is retained while the binary assumption is discarded. In other words, we take into account the term frequency of each term in each document. Then the optimal weight of a term  $T_j$  with frequency of occurrences  $d_{ij}$  in a document  $D_i$  is calculated by the following formula [YuLe].

$$w_2(T_j, d_{ij}) = \log \frac{p(tf=d_{ij}|R)}{p(tf=d_{ij}|I)} - \log \frac{p(tf=0|R)}{p(tf=0|I)} \quad (2.2)$$

where  $p(tf=d_{ij}|R)$  is the probability that a relevant document has  $d_{ij}$  occurrences of the term  $T_j$ , and  $p(tf=d_{ij}|I)$  represents the probability that an irrelevant document has  $d_{ij}$  occurrences of the term  $T_j$ . "R" and "I" stand for "relevance" and "irrelevance", respectively.  $p(tf=0|R)$  and  $p(tf=0|I)$  are the probabilities with 0 occurrence of the term. For a query

$$Q = (T_{k1}, T_{k2}, \dots, T_{kl})$$

and a document

$$D_i = (d_{i1}, d_{i2}, \dots, d_{iM})$$

the similarity between  $Q$  and  $D_i$  is

$$\text{sim}(Q, D_i) = \sum_{n=1}^l \delta_{kn} \cdot w_2(T_{kn}, d_{kn})$$

where  $\delta_{kn}$  is one when  $d_{kn} \neq 0$ , and zero when  $d_{kn} = 0$ .

Again the probabilities  $p(tf=d_{ij}|R)$  and  $p(tf=d_{ij}|I)$  are usually unknown in advance. In addition to the estimation of  $p_j$  and  $q_j$ , the probabilities  $p(tf=d_{ij}|R)$  and  $p(tf=d_{ij}|I)$  are estimated from another available parameter in this paper.

### 3. Parameter Estimation of Two Independence Models

In order to estimate the probability values of the two independence models described in 2.2 and 2.3, we suppose that the probability values are correlated with document frequencies. Document frequency  $n_j$  is the number of documents which have term  $T_j$ . It seems to be a reasonable assumption that the probability values of a term  $T_j$  increases with its document frequency  $n_j$ .

Since document frequency  $n_j$  is a parameter which is easily obtainable, it becomes easy to predict the probability values in advance if a simple relationship between the probability values and document frequencies does exist.

An attempt to estimate parameters for the binary independence model has been made before[CrHa]. The estimation method given below is not only for the binary model but also for the non-binary model, and is more elaborate than the previous attempt.

#### 3.1. Parameter Estimation of Binary Independence Model

**Hypothesis I** The probability values  $p_j$  and  $q_j$  in (2.1) are linearly correlated with the document frequency  $n_j$  of term  $T_j$ . In other words, there exist linear functions  $EP_1$  and  $EQ_1$  such that

$$p_j = EP_1(n_j), \quad q_j = EQ_1(n_j).$$

Basing on the hypothesis I, we try to find the functions  $EP_1$  and  $EQ_1$  by the following steps.

**Preconditions:** We have a collection of documents  $D$  and a set of queries  $Q$ . For each query of  $Q$ , the set of relevant documents is known beforehand.

**Step 1 : [Partition of queries]** Divide a set of queries  $Q$  into two sets of queries  $Q_1$  and  $Q_2$  by some criterion. The query set  $Q_1$  is used to estimate the probability values  $p_j$  and  $q_j$  from the document frequency  $n_j$ , while  $Q_2$  is used to evaluate the performance of the estimated binary independence model. The following criterion is used to partition  $Q$  into  $Q_1$  and  $Q_2$  when it is applicable.

criterion1:

Divide  $Q$  in such a way that each term of each query in  $Q_2$  is contained in some query of  $Q_1$ .

If criterion 1 can not be satisfied by the queries in the collections, then use the following second criterion.

criterion2:

According to the empirical rule(the 2/3, 1/3 split)[Brei],  $Q_1$  should contain 2/3 of all queries and  $Q_2$  should contain the remaining 1/3 of queries.

**Step 2 : [Data Plot]** Count  $n_j$ (a document frequency),  $r_j$ (the number of relevant documents having term  $T_j$ ), and  $s_j$ (the number of irrelevant documents having term  $T_j$ ) for each term  $T_j$  of each query in  $Q_1$ . Plot the points  $(n_j, r_j/R)$  and  $(n_j, s_j/I)$  in each scattergram, where  $R$  is the total number of relevant documents and  $I$  is the total number of irrelevant documents. For a document frequency which has more than one corresponding  $r_j/R$  (or  $s_j/I$ ), calculate their arithmetic mean and plot it. Having done this, we obtain two scattergrams, one for  $EP_1$  and the other for  $EQ_1$ .

**Step 3 : [Estimation of  $EP_1$  and  $EQ_1$ ]** Suppose that  $EP_1$  and  $EQ_1$  are linear functions of document frequencies, that is to say,

$$\begin{aligned} EP_1(n_j) &= a + bn_j, \\ EQ_1(n_j) &= c + dn_j. \end{aligned}$$

Then calculate the regression coefficients  $a, b, c$ , and  $d$  by solving the following normal equations.

$$\sum_{j=1}^{N_0} \frac{r_j}{R} = aN_0 + b \sum_{j=1}^{N_0} n_j \quad \sum_{j=1}^{N_0} n_j \frac{r_j}{R} = a \sum_{j=1}^{N_0} n_j + b \sum_{j=1}^{N_0} n_j^2$$

for  $a$  and  $b$ , and

$$\sum_{j=1}^{N_0} \frac{s_j}{I} = cN_0 + d \sum_{j=1}^{N_0} n_j \quad \sum_{j=1}^{N_0} n_j \frac{s_j}{I} = c \sum_{j=1}^{N_0} n_j + d \sum_{j=1}^{N_0} n_j^2$$

for  $c$  and  $d$ , where  $N_0$  is the number of data points in the scattergrams obtained in step 2. Standard error of estimate  $s_{ep}^2$  and  $s_{eq}^2$  are calculated by the formula,

$$s_{ep}^2 = \frac{1}{N_0-2} \sum_{j=1}^{N_0} \left[ \frac{r_j}{R} - (a+bn_j) \right]^2$$

$$s_{eq}^2 = \frac{1}{N_0-2} \sum_{j=1}^{N_0} \left[ \frac{r_j}{I} - (c+dn_j) \right]^2$$

*Step 4 : [Modification of  $EP_1$  and  $EQ_1$ ]* We obtain the estimations of  $EP_1$  and  $EQ_1$  in the previous step. These estimations may be crude and have to be refined by some rules, because under some situations enough data may not be available to make accurate estimations. The constraints defined below are from reasonable assumptions about probabilities and weighting functions.

(A0)  $EP_1$  and  $EQ_1$  should be non-negative increasing functions of document frequencies.

(A1) At the total number of documents  $N$ ,  $EP_1$  and  $EQ_1$  must be one.

(A2) A weighting function derived from  $EP_1$  and  $EQ_1$  should be a decreasing function of document frequencies.

From the constraint A0, we get the first condition for the regression coefficients  $a, b, c$ , and  $d$  as follows.

$$a \geq 0, \quad b > 0, \quad c \geq 0, \quad d > 0 \quad (3.1)$$

From the constraint A1, the second condition satisfied by the regression coefficients is obtained.

$$EP_1(N) = a+bN = 1, \quad EQ_1(N) = c+dN = 1. \quad (3.2)$$

The estimated weighting function of the binary independence model is

$$EW(n) = \log \frac{a+bn}{1-a-bn} - \log \frac{c+dn}{1-c-dn}$$

Calculating the derivative of  $EW(n)$ , we get the sufficient conditions for the regression coefficients to satisfy the constraint A2 as follows.

$$d > b, \quad (b-d) \left( \frac{a(1-a)}{b} - \frac{c(1-c)}{d} \right) \geq (a-c)^2 \quad (3.3)$$

We have to pay attention to the fact that the condition (3.3) is not a necessary but sufficient condition, therefore we may disregard this condition as far as we make sure that  $EW(n)$  is a decreasing function in the range of concern ( $0 < n < N$ ).

*Step 5 : [Evaluation of the estimations]* Measure the performance of the estimated binary independence model in terms of recall and precision by using the query set  $Q_2$ .

Recall and precision are defined by

$$\text{Recall} = \frac{\text{the number of relevant documents retrieved}}{\text{the total number of relevant documents}}$$

$$\text{Precision} = \frac{\text{the number of relevant documents retrieved}}{\text{the number of documents retrieved}}$$

A weight of a term  $T_j$  in a query  $Q_2$  is calculated by the estimated weighting function obtained in the step 3 and step 4 as follows.

$$w_1(T_j) = EW(n_j) = \log \frac{EP_1(n_j)}{1-EP_1(n_j)} - \log \frac{EQ_1(n_j)}{1-EQ_1(n_j)}$$

where  $n_j$  is a document frequency of a term  $T_j$ . In order to evaluate performance of the estimated binary independence model, the results of retrieval are compared to the inverse document frequency method, the cosine measure, and the statistical similarity measure.

### 3.2. Parameter Estimation of Non-Binary Independence Model

**Hypothesis II** The probability values  $p(tf=d_{ij} | R)$  and  $p(tf=d_{ij} | I)$  in (2.2) are correlated with document frequency  $n_j$  and term frequency  $d_{ij}$ . In other words, there exist some linear functions  $EP_2$  and  $EQ_2$  such that

$$p(tf=d_{ij} | R) = EP_2(n_j, d_{ij}),$$

$$p(tf=d_{ij} | I) = EQ_2(n_j, d_{ij}).$$

Then we try to estimate  $EP_2$  and  $EQ_2$  by the following steps.

Preconditions and step 1 are the same as in 3.1.

**Step 2 : [Plot of Data]** Count  $n_j$ ,  $r_{jt}$  (the number of relevant documents which have term  $T_j$  of term frequency  $t$ ), and  $s_{jt}$  (the number of irrelevant documents which have term  $T_j$  of term frequency  $t$ ) for each term  $T_j$  of each query in the query set  $Q_1$ . Plot the points  $(n_j, r_{jt}/R)$ , and  $(n_j, s_{jt}/I)$ . For a document frequency and a term frequency which have more than one corresponding  $r_{jt}/R$  ( or  $s_{jt}/I$  ), calculate their arithmetic mean and plot it. Having done this, we obtain a number of scattergrams, each of which shows the relation of  $n_j$  to  $r_{jt}/R$  ( or  $s_{jt}/I$  ) of the term frequency  $t$ .

**Step 3 : [Estimations of  $EP_2$  and  $EQ_2$ ]** Based on the scattergrams obtained in step 2, estimate  $EP_2$  and  $EQ_2$  by the linear regression.

$$EP_2(n_j, t) = a_t + b_t n_j$$

$$EQ_2(n_j, t) = c_t + d_t n_j$$

where  $t$  is a term frequency ranging from 1 to  $F$ , and  $F$  is the maximum number of a term frequency. Each set of regression coefficients  $a_t$ ,  $b_t$ ,  $c_t$ , and  $d_t$  are calculated by the same kind of normal equations in step 3 of 3.1.

**Step 4 : [Modifications of  $EP_2$  and  $EQ_2$ ]**  $EP_2$ 's and  $EQ_2$ 's obtained in step 3 may not be accurate because we do not have enough data to make accurate estimations under usual circumstances. Therefore we need the step to compensate the scarcity of data in order to make the estimations more accurate. As in the step 4 of 3.1, we set up some constraints which are supposed to be satisfied by  $EP_2$ 's and  $EQ_2$ 's: Using the constraints, we modify the regression coefficients  $a_t$ ,  $b_t$ ,  $c_t$ , and  $d_t$ .

- (B0)  $EP_2$ 's and  $EQ_2$ 's are non-negative increasing functions of document frequencies.  
 (B1) Sum of  $EP_2$ 's is equal to  $EP_1$ , and sum of  $EQ_2$ 's is equal to  $EQ_1$ . That is to say,

$$EP_1(n) = \sum_{t=1}^F EP_2(n, t), \quad EQ_1(n) = \sum_{t=1}^F EQ_2(n, t).$$

where  $F$  is the maximum number of term frequencies.

- (B2) A term of low term frequency is usually more common than a term of high term frequency. In other words,

$$EP_2(n, t) < EP_2(n, t-1), \quad EQ_2(n, t) < EQ_2(n, t-1), \quad (t > 1).$$

- (B3) Weighting functions derived from  $EP_2$ 's and  $EQ_2$ 's should be decreasing functions of document frequencies.

From the rule B0, B1, and B2, the following conditions for the regression coefficients are derived,

$$a_t \geq 0, \quad b_t > 0, \quad c_t \geq 0, \quad d_t > 0 \quad (3.4)$$

$$a = \sum_{t=1}^F a_t, \quad b = \sum_{t=1}^F b_t, \quad c = \sum_{t=1}^F c_t, \quad d = \sum_{t=1}^F d_t. \quad (3.5)$$

$$a_{t-1} \geq a_t, \quad b_{t-1} > b_t, \quad c_{t-1} \geq c_t, \quad d_{t-1} > d_t. \quad (3.6)$$

From the rule B3, we obtain the following sufficient conditions for a monotonic decreasing property of weighting functions.

$$\left(1 + \frac{b-d-bc-ad}{bd}\right) b_t c_t < a_t b_t, \quad (3.7)$$

$$[(b-c-A)(a_t d_t + b_t c_t) - A_t(b+d-bc-ad)]^2 < 4(A_t b d + (b-d-A) B_t d_t)(A_t(1-a-c) + (b-d-A) a_t c_t)$$

where  $A = bc - ad$  and  $A_t = b_t c_t - a_t d_t$ . Again, since (3.7) is not a necessary condition but a sufficient one, we may discard (3.7) as far as we make sure that weighting functions are decreasing functions in the range of our concern.

**Step 5 : [Evaluation of the estimations]** As the same as the step 5 of 3.1, measure the performance of the estimated non-binary independence model in terms of recall and precision. A weight of a term  $T_j$  occurring  $t$  times in a document is calculated by the following formula.

$$w_2(T_j, t) = EW_t(n_j) = \log \frac{EP_2(n_j, t)}{EQ_2(n_j, t)} - \log \frac{1-EP_1(n_j)}{1-EQ_1(n_j)}$$

The performance is compared to that of the inverse document frequency method, the cosine measure, and the statistical similarity measure.

#### 4. Experiments

Three different document collections are used to evaluate the performance of the two estimated independence models. The document collections are called CISI, MEDLARS, and CRN4NUL, respectively.

##### 4.1. Preliminary Statistics of Document Collections

The following table gives some statistics of three document collections used in the experiments.

Collection	CISI	MEDLARS	CRN4NUL
N	1460	1033	424
Q	76	30	155
M	7737	6928	2652
AMD	100	51	53
AMQ	28	10	8

In the above table, the meanings of the symbols are as follows.

N	total number of documents
Q	total number of queries
M	total number of terms
AMD	average number of terms contained in a document
AMQ	average number of terms contained in a query

##### 4.2. Results of Estimated Binary Independence Model

The criterion 1 of the query partition can be applied to CISI and CRN4NUL collections. For MEDLARS, the criterion 2 is applied. The results of the query partitions are shown in Table 4.2. The numbers of queries contained in each set of queries are shown in the table.

Collection	CISI	MEDLARS	CRN4NUL
$Q_1$	61	20	110
$Q_2$	15	10	45

The regression coefficients  $a$ ,  $b$ ,  $c$ , and  $d$  for each collection are obtained by using the query set  $Q_1$  as in the following table. The scattergrams and the linear regressions of CRN4NUL collection are shown in Fig. 4.1.

Collection	CISI	MEDLARS	CRN4NUL
$a$	0.04209	0.05437	0.07145
$b$	0.00089	-0.00021	-0.00034
$c$	-0.00054	-0.00140	-0.00113
$d$	0.00068	0.001	0.00240

Since the coefficient  $c$  of every collection is negative and does not satisfy (3.1),  $c$  is modified to 0. Then, the second condition of (3.2) becomes  $dN = 1$ . The first condition of (3.2) is not satisfied by the coefficients  $a$  and  $b$  of every collection as follows.

$$EP_1(N) = a + bN = 0.04209 + 0.00089 \times 1460 = 1.3415 \quad (\text{CISI}),$$

$$EP_1(N) = a + bN = 0.05437 - 0.00021 \times 1033 = -0.1626 \quad (\text{MEDLARS}),$$

$$EP_1(N) = a + bN = 0.07145 - 0.00034 \times 424 = -0.07271 \quad (\text{CRN4NUL}).$$

Denoting the modified coefficients by  $a'$ ,  $b'$ ,  $c'$ , and  $d'$ , we obtain the constraints for these coefficients as follows.

$$a' = 1 - b'N \quad c' = 0 \quad d' = \frac{1}{N} \quad (4.1)$$

where  $N$  is the total number of documents. If we use (4.1) to choose the regression coefficients, then it becomes impossible to satisfy (3.3) which is derived from the rule A2, because R.H.S. of the second inequality of (3.3) is

$$(a' - c')^2 = a'^2.$$

On the other hand, its L.H.S. becomes

$$-(b' - d') \left( \frac{c'(1-c')}{d'} - \frac{a'(1-a')}{b'} \right) = -a'^2.$$

Since this condition is sufficient, we can use (4.1) as far as we make sure that a weighting function derived from (4.1) is a decreasing function as document frequency increases in the range of our concern.

We still have ambiguity in deciding  $a'$  and  $b'$ . If we rely on  $a$  rather than  $b$ , then  $b'$  must be  $(1-a)/N$ , in order to satisfy the first condition of (3.2). Here we bring in an empirical rule, that is to say, a ratio of  $d$  to  $d'$  is equal to a ratio of  $(1-a)/N$  to  $b'$ . In other words,

$$\frac{1-a}{N} : b' = d : d'.$$

This means that if we interpret  $(1-a)/N$  as an experimental result for gradient of  $p_j$ , then ratio of gradients of  $p_j$  and  $q_j$  before and after the modification must be equal. Now using the rule, we decide  $a'$  and  $b'$  by the following formula.

$$b' = \frac{1-a}{N^2 d} \quad a' = 1 - b'N \quad (4.2)$$

Even though (4.2) does not give any optimal choice of  $a'$  and  $b'$ , and other choices of  $a'$  and  $b'$  may give better performance than (4.2), (4.2) seems to be a reasonable choice.

The regression coefficients modified by (4.1) and (4.2) are as follows.

Collection	CISI	MEDLARS	CRN4NUL
$a'$	0.03494	0.0796	0.0892
$b'$	0.000661	0.000891	0.002148
$c'$	0.0	0.0	0.0
$d'$	0.000685	0.000968	0.00236

The estimated weighting functions for three collections derived from Table 4.4 are shown in Fig.4.2. The performance of the estimated binary independence model using the regression coefficients of Table 4.4 is measured in terms of recall and precision. The results are shown in Table 4.5 with the performances of the other three methods. In Table 4.5, all numbers appeared are average precisions over the queries contained in  $Q_2$ . Average improvements of the estimated binary independence model compared to the other three methods are shown in Table 4.6. This table shows that the estimated binary independence model retrieves documents, for instance, 4.9 % better than the inverse document frequency method, and 26.5 % better than the cosine measure in CISI collection. Note that the results are reported for query set  $Q_2$  only, where parameter estimation is not performed.

recall	eBIM	IDFM	COSINE	SSM
0.1	0.3670	0.3135	0.2330	0.2379
0.2	0.2776	0.2637	0.1665	0.1906
0.3	0.2075	0.1979	0.1541	0.1628
0.4	0.1639	0.1680	0.1323	0.1391
0.5	0.1477	0.1343	0.1172	0.1216
0.6	0.1265	0.1162	0.1042	0.1060
0.7	0.1093	0.1063	0.0924	0.0953
0.8	0.0937	0.0945	0.0834	0.0811
0.9	0.0762	0.0739	0.0716	0.0672
1.0	0.0471	0.0473	0.0484	0.0478

### 4.3. Results of Estimated Non-Binary Independence Model

The regression coefficients  $a_i$ ,  $b_i$ ,  $c_i$ , and  $d_i$  estimated for each collection by using  $Q_1$  are shown in Table 4.7. In this table, "tf" means a term frequency and "pts" means the number of data points used to estimate the regression coefficients for each term frequency. Scattergrams and linear regressions of CRN4NUL collection are shown in Fig. 4.3.

recall	eBIM	IDFM	COSINE	SSM
0.1	0.8261	0.8241	0.8595	0.8700
0.2	0.7898	0.8064	0.7133	0.7678
0.3	0.6948	0.6747	0.5424	0.6454
0.4	0.6521	0.6476	0.5039	0.5237
0.5	0.5340	0.5560	0.4210	0.4554
0.6	0.4362	0.4329	0.3718	0.4226
0.7	0.3462	0.3157	0.3204	0.3413
0.8	0.2796	0.2755	0.2676	0.2756
0.9	0.1811	0.1781	0.1858	0.1911
1.0	0.0651	0.0842	0.1189	0.1048

recall	eBIM	IDFM	COSINE	SSM
0.1	0.6174	0.6464	0.5865	0.6449
0.2	0.5054	0.4998	0.4352	0.4903
0.3	0.4034	0.3865	0.3675	0.4014
0.4	0.3548	0.3290	0.2867	0.2978
0.5	0.3007	0.2863	0.2341	0.2432
0.6	0.2520	0.2515	0.2026	0.2200
0.7	0.2026	0.2015	0.1658	0.1826
0.8	0.1607	0.1577	0.1441	0.1510
0.9	0.1377	0.1350	0.1204	0.1289
1.0	0.1261	0.1246	0.1131	0.1199

Collection	CISI	MEDLARS	CRN4NUL
IDFM	4.9	-1.1	2.0
COSINE	26.5	7.4	16.7
SSM	22.8	1.1	8.6

tf	$a_t$	$b_t$	$c_t$	$d_t$	pts
1	0.04851	0.00034	0.01382	0.00036	191
2	-0.00022	0.00024	-0.00366	0.00014	188
3	0.00535	0.00010	-0.00268	0.00007	181
4	0.00342	0.00005	-0.00217	0.00004	157
5	0.01432	-0.00001	-0.00141	0.00003	126
6	0.00515	0.00001	-0.00119	0.00002	84
7	0.00413	0.00001	-0.00059	0.00001	70
8	0.01004	-0.00001	-0.00022	0.00001	45
9	0.00102	0.00001	0.00007	0.00001	35
10	0.02001	-0.00005	0.00027	0.0	28
11	0.00078	0.00001	0.00059	0.0	13
12	0.03193	-0.00008	0.00004	0.0	7
13	0.01193	0.00003	-0.00019	0.0	9

Using the rules B0, B1, B2, and B3, we modify the regression coefficients of Table 4.7 and make a modified set of regression coefficients as shown in Table 4.8. First of all, the regression coefficients estimated by less than 20% of maximum pts are discarded, because we can not expect enough reliability in such data. In order to decide the coefficients uniquely, the following set of formula is used in addition to (3.4)-(3.7).

$$c''_t = 0 \quad d''_t = \frac{Nd_t + c_t}{N} \quad b''_t = \frac{Nd''_t - a_t}{N} \quad a''_t = a_t \quad (4.3)$$

The first condition of (4.3) is derived from (3.4), (3.5), and (4.1). (3.4) and (3.5) are

$$c'_t \geq 0 \quad , \quad c' = \sum_{t=1}^F c'_t$$

On the other hand,  $c'$  is 0 by (4.1). Therefore all  $c'_t$ 's must be 0. The second condition of (4.3) is based on the fact that probability values  $p(tf = d_{ij} | I)$  at  $N$  seem to be reliable because

$$\begin{aligned} \sum_{t=1}^8 (c_t + Nd_t) &= 0.9801 \approx 1 \quad (CISI), \\ \sum_{t=1}^9 (c_t + Nd_t) &= 1.0003 \approx 1 \quad (MEDLARS), \\ \sum_{t=1}^8 (c_t + Nd_t) &= 0.9894 \approx 1 \quad (CRN4NUL). \end{aligned}$$



tf	$a_t$	$b_t$	$c_t$	$d_t$	pts
1	0.08776	0.00073	0.01131	0.00123	101
2	0.08743	0.00030	-0.00301	0.00054	101
3	0.07063	-0.00013	-0.00267	0.00026	98
4	0.05260	-0.00011	-0.00070	0.00013	87
5	0.06180	-0.00031	0.00040	0.00007	77
6	0.03777	-0.00019	0.00107	0.00004	62
7	0.03653	-0.00012	0.00117	0.00003	46
8	0.04788	-0.00021	0.00239	0.00001	36
9	0.03432	-0.00010	0.00120	0.00002	17
10	0.02545	-0.00007	0.00200	0.00001	17
11	0.10018	-0.00076	0.00146	0.00001	10
12	0.02234	-0.00016	0.00202	0.0	5
13	0.07231	-0.00038	0.00121	0.00001	5
14	0.03541	-0.00052	0.00154	0.00001	3

tf	$a_t$	$b_t$	$c_t$	$d_t$	pts
1	0.07478	0.00027	-0.0007	0.00059	93
2	0.05337	0.00020	-0.00194	0.00020	92
3	0.04951	-0.00006	-0.00051	0.00008	90
4	0.04028	-0.00007	0.00002	0.00004	77
5	0.04313	-0.00014	0.00030	0.00002	60
6	0.03243	-0.00008	0.00055	0.00001	60
7	0.04757	-0.00017	-0.00045	0.00002	40
8	0.03423	-0.00012	0.00030	0.00001	24
9	0.02769	-0.00008	0.00072	0.0	21
10	0.00941	0.00001	0.00105	0.0	13
11	0.00508	0.00001	0.00096	0.0	15
12	0.03935	-0.00021	0.00015	0.00001	13
13	-0.06018	0.00074	0.00331	-0.00003	3

Taking  $c_t + Nd_t$  as the actual value of  $p(tf = d_{ij} | I)$  at  $N$ , we obtain the following equation.

$$c''_t + Nd''_t = Nd''_t = c_t + Nd_t$$

This leads to the second condition of (4.3). The third condition of (4.3) is based on the assumption that the probability values of  $p(tf = d_{ij} | R)$  and  $p(tf = d_{ij} | I)$  are the same at  $N$ . This means that a proportion of a probability value of each term frequency to the total (total is 1) at  $N$  is the same in  $p(tf = d_{ij} | I)$  and  $p(tf = d_{ij} | R)$ . Again we rely on  $a_t$ 's rather than  $b_t$ 's, then  $b''_t$  must satisfy

$$a_t + Nb''_t = c''_t + Nd''_t = Nd''_t$$

If one of the following things happened, then  $a''_t$  or  $b''_t$  or  $d''_t$  are set half of  $a''_{t-1}$  or  $b''_{t-1}$  or  $d''_{t-1}$ , respectively. This rule is obtained by observing the experimental results of  $d_t$ , that is  $d_t \approx 0.5d_{t-1}$ .

- $a''_t < 0$  or  $b''_t < 0$  or  $d''_t < 0$
- $a''_t > a''_{t-1}$  or  $b''_t > b''_{t-1}$  or  $d''_t > d''_{t-1}$
- $a''_t < 0.1a''_{t-1}$  or  $b''_t < 0.1b''_{t-1}$  or  $d''_t < 0.1d''_{t-1}$

In order to satisfy (3.5), the final coefficients used to calculate weights are given by

$$a'_t = \frac{a''_t}{\sum_{k=1}^F a''_k} \quad b'_t = \frac{b''_t}{\sum_{k=1}^F b''_k} \quad c'_t = 0.0 \quad d'_t = \frac{d''_t}{\sum_{k=1}^F d''_k} \quad (4.4)$$

So far, since we do not consider B3, we have to make sure that weighting functions derived from (4.3) and (4.4) are decreasing functions of document frequency. Again, although (4.3) and (4.4) do not guarantee any optimality of choices, it usually gives reasonable performances. The weighting functions derived from Table 4.8(c) are shown in Fig. 4.4.

tf	$a'_t$	$b'_t$	$c'_t$	$d'_t$
1	0.020001	0.000359	0.0	0.000374
2	0.010001	0.000147	0.0	0.000139
3	0.002206	0.000069	0.0	0.000069
4	0.001410	0.000039	0.0	0.000039
5	0.000705	0.000021	0.0	0.000029
6	0.000353	0.000017	0.0	0.000019
7	0.000176	0.000007	0.0	0.000010
8	0.000088	0.000004	0.0	0.000005

tf	$a'_t$	$b'_t$	$c'_t$	$d'_t$
1	0.026152	0.000634	0.0	0.00060
2	0.018665	0.000180	0.0	0.00020
3	0.017315	0.000039	0.0	0.00008
4	0.008754	0.000019	0.0	0.00004
5	0.004478	0.000010	0.0	0.00002
6	0.002379	0.000005	0.0	0.000011
7	0.001189	0.000002	0.0	0.000005
8	0.000595	0.0000012	0.0	0.000003
9	0.000074	0.0000006	0.0	0.000001

tf	$a'_t$	$b'_t$	$c'_t$	$d'_t$
1	0.023502	0.001457	0.0	0.00127
2	0.023414	0.000453	0.0	0.00054
3	0.018915	0.000121	0.0	0.00026
4	0.009627	0.000060	0.0	0.00013
5	0.005582	0.000030	0.0	0.00007
6	0.003592	0.000015	0.0	0.00004
7	0.003102	0.000008	0.0	0.00003
8	0.001466	0.000004	0.0	0.00002

recall	eNBIM	IDFM	COSINE	SSM
0.1	0.3784	0.3135	0.2330	0.2379
0.2	0.2923	0.2637	0.1665	0.1906
0.3	0.2046	0.1979	0.1541	0.1628
0.4	0.1771	0.1680	0.1323	0.1391
0.5	0.1502	0.1343	0.1172	0.1216
0.6	0.1324	0.1162	0.1042	0.1060
0.7	0.1111	0.1063	0.0924	0.0953
0.8	0.0992	0.0945	0.0834	0.0811
0.9	0.0785	0.0739	0.0716	0.0672
1.0	0.0481	0.0473	0.0484	0.0478

recall	eNBIM	IDFM	COSINE	SSM
0.1	0.8397	0.8241	0.8595	0.8700
0.2	0.8175	0.8064	0.7133	0.7678
0.3	0.7685	0.6747	0.5424	0.6454
0.4	0.6443	0.6476	0.5039	0.5237
0.5	0.5557	0.5560	0.4210	0.4554
0.6	0.4473	0.4329	0.3718	0.4226
0.7	0.3940	0.3157	0.3204	0.3413
0.8	0.2887	0.2755	0.2676	0.2756
0.9	0.1494	0.1781	0.1858	0.1911
1.0	0.0764	0.0842	0.1189	0.1048

recall	eNBIM	IDFM	COSINE	SSM
0.1	0.6449	0.6464	0.5865	0.6449
0.2	0.5444	0.4998	0.4352	0.4903
0.3	0.4501	0.3865	0.3675	0.4014
0.4	0.3935	0.3290	0.2867	0.2978
0.5	0.3104	0.2863	0.2341	0.2432
0.6	0.2586	0.2515	0.2026	0.2200
0.7	0.2052	0.2015	0.1658	0.1826
0.8	0.1561	0.1577	0.1441	0.1510
0.9	0.1262	0.1350	0.1204	0.1289
1.0	0.1155	0.1246	0.1131	0.1199

Collection	CISI	MEDLARS	CRN4NUL
IDFM	8.3	2.4	4.2
COSINE	30.8	11.0	19.4
SSM	27.0	4.4	11.0

The performance of the estimated non-binary independence model using the regression coefficients of Table 4.8 is shown in Table 4.9 with the performances of the other three methods. The numbers appearing in Table 4.9 are the average precisions over the queries contained in  $Q_2$ . Improvements of the estimated non-binary independence model compared to the other three methods are shown in Table 4.10. This table shows that the estimated non-binary independence model retrieves documents, for instance, 8.3% better than the inverse document frequency method, and 30.8% better than the cosine measure in CISI collection.

## 5. Motivations of the Second Method

The first method may be unable to make the most of relevance information. For instance, the method can not distinguish terms of the same document frequency and/or term frequency. Another disadvantage is that we have to bring in some empirical rules to determine the coefficients uniquely. In order to compensate these disadvantages, a formula is given to learn optimal weights of the non-binary independence model. The formula is applied to the same three document collections. Effects of a coefficient used in the formula and the number of learning times on retrieval performances are measured in terms of average improvements compared to the inverse document frequency method.

## 6. Learning Scheme

### 6.1. Learning Formula

A learning formula used in the experiments is as follows.

$$w_{k+1}(T_j, tf) = w_k(T_j, tf) + \frac{c}{n_j} [w_{opt}(T_j, tf) - w_k(T_j, tf)] \quad (6.1)$$

where  $w_k(T_j, tf)$  is a weight of  $j^{th}$  term  $T_j$  with term frequency  $tf$  after learning  $k$  times,  $w_{opt}(T_j, tf)$  is optimal weight of  $j^{th}$  term  $T_j$  with term frequency  $tf$  by the non-binary independence model,  $c$  is a learning coefficient, and  $n_j$  is document frequency of term  $T_j$ . Although  $w_{opt}(T_j, tf)$  is determined not only by  $T_j$  and  $tf$  but also by a query in a learning query set, we do not express it explicitly here for simplicity of expression.  $w_k(T_j, tf)$  and  $w_{opt}(T_j, tf)$  will be abbreviated as  $w_k$  and  $w_{opt}$  in the sequel.

The reason why document frequency  $n_j$  is incorporated in this formula is that weights of term  $T_j$  are learned  $m \times n_j$  times at every iteration of learning, where  $m$  is the number of occurrences of term  $T_j$  in the learning query set. This means that weights of high document frequency terms are learned many more times than weights of low document frequency term. Document frequency  $n_j$  in the formula (6.1) prevents this inequality in learning.

### 6.2. Properties of Learning Formula

It can be easily derived from (6.1) that the formula expressing  $w_k$  by  $w_0$  (initial weight) is as follows.

$$w_k = [1 - (1 - \frac{c}{n_j})^k] (w_{opt} - w_0) + w_0. \quad (6.2)$$

From (6.2), we recognize that if  $\frac{c}{n_j}$  is close to one, then  $(1 - \frac{c}{n_j})^k$  rapidly approaches to zero as  $k$  increases, and this means  $w_k$  rapidly approaches to  $w_{opt}$  with times of learning.

If  $\frac{c}{n_j}$  is very small and  $k$  is also small, then the following approximation holds.

$$(1 - \frac{c}{n_j})^k \approx 1 - \frac{kc}{n_j}.$$

And (6.2) becomes

$$w_k \approx w_0 + \frac{kc}{n_j} (w_{opt} - w_0).$$

This means that  $k$  steps of learning using  $c$  is almost equivalent to one step of learning using  $kc$ , provided  $\frac{c}{n_j}$  is very small and  $k$  is also small.

This relation of learning coefficients and the number of learning steps will be observed in the results of the experiments in section 7.

If  $w_{opt}$  is unique, then  $w_k$  never overshoots or undershoots  $w_{opt}$  as far as  $\frac{c}{n_j}$  is less than one. However, as pointed out in 6.1, the values of  $w_{opt}$  may be different for different query, and this may cause  $w_k$  to overshoot or undershoot some values of  $w_{opt}$ .

### 6.3 Learning Procedure

The following is the learning procedure used in the experiments.

- 1° Assign a weight by the inverse document frequency method to each query term contained in a learning query set as an initial weight.
- 2° Using (6.1) and the learning query set, modify weights in documents.

- 3° Using another query set, evaluate performance of the weights modified in 2°. Compare the performance of the weights to the performance of the inverse document frequency method.
- 4° Go to 2° until the learning is executed sufficient times.

The following example illustrates the learning procedure described above.

**Example** Suppose that term  $T_1$  has the following distribution of term frequencies in total ten documents.

Term-Doc	$D_1$	$D_2$	$D_3$	$D_4$	$D_5$	$D_6$	$D_7$	$D_8$	$D_9$	$D_{10}$
$T_1$	0	1	2	0	0	1	0	1	2	0

Then initial weights of documents are

Term-Doc	$D_1$	$D_2$	$D_3$	$D_4$	$D_5$	$D_6$	$D_7$	$D_8$	$D_9$	$D_{10}$
$T_1$	0	$\log 2$	$\log 2$	0	0	$\log 2$	0	$\log 2$	$\log 2$	0

Suppose that  $D_{1-6}$  are non-relevant and  $D_{7-10}$  are relevant to the example query. Then

$$w_{opt}(T_1,1) = \log \frac{3}{4} \approx -0.29,$$

$$w_{opt}(T_1,2) = \log \frac{3}{2} \approx 0.4.$$

$w_k(T_1,1)$  is learned three times, and  $w_k(T_1,2)$  is learned two times during one iteration of learning (i.e. going through all documents having the term). Therefore during first time of learning with coefficient 0.2, the weights are changed after one iteration of learning as follows.

$$w_1(T_1,1) = \log 2 + \frac{0.2}{5} (\log \frac{3}{4} - \log 2) = 0.65,$$

$$w_2(T_1,1) = 0.65 + \frac{0.2}{5} (\log \frac{3}{4} - 0.65) = 0.61,$$

$$w_3(T_1,1) = 0.61 + \frac{0.2}{5} (\log \frac{3}{4} - 0.61) = 0.57.$$

And

$$w_1(T_1,2) = \log 2 + \frac{0.2}{5} (\log \frac{3}{2} - \log 2) = 0.68,$$

$$w_2(T_1,2) = 0.68 + \frac{0.2}{5} (\log \frac{3}{2} - 0.68) = 0.67.$$

The weights in the documents are changed as follows.

Term-Doc	$D_1$	$D_2$	$D_3$	$D_4$	$D_5$	$D_6$	$D_7$	$D_8$	$D_9$	$D_{10}$
$T_1$	0	0.57	0.67	0	0	0.57	0	0.57	0.67	0

□

## 7. Experimental Results of the Second Method

The experiments are performed on three different document collections, CISI, MEDLARS, and CRN4NUL. The query sets for learning and evaluation are the same as those in the experiments of section 4, that is to say,  $Q_1$  and  $Q_2$ , respectively.

Five different learning coefficients ranging from 0.002 to 0.1 are experimented. Learning curves of each learning coefficient are shown in Fig.7.1. The improvement in the figure is measured compared to the inverse document frequency method. The precision-recall graph of the learning formula after 10 times of learning with coefficient 0.016 in CISI collection is shown in Fig.7.2 with the precision-recall of the inverse document frequency method.

Table 7.1 shows recall-precision of the formula after each time of learning in CISI. From this table, we can observe that one step of learning with coefficient 0.016 is almost equal to two steps of learning with coefficient 0.008 as pointed out in section 6.

Table 7.2 shows average improvements of the scheme using coefficient 0.016 after 10 times of learning. The improvements are measured compared to the inverse document frequency method.

CISI	MEDLARS	CRN4NUL
7.8	1.0	7.3

Table 7.1(a) Precision-Recall of CISI(c=0.008)											Table 7.1(b) Precision-Recall of CISI(c=0.016)										
precision improvement compared to IDFM (%)											precision improvement compared to IDFM (%)										
recall-step	1	2	3	4	5	6	7	8	9	10	recall-step	1	2	3	4	5	6	7	8	9	10
0.1	1.3	1.9	3.1	3.4	3.7	4.3	3.0	1.6	1.6	3.1	0.1	1.9	3.7	4.3	1.4	3.1	4.7	5.4	6.5	8.9	10.1
0.2	3.8	5.7	7.2	6.9	7.2	7.8	8.5	9.6	9.9	10.3	0.2	5.7	7.2	7.4	9.3	10.5	9.3	11.7	11.1	11.6	12.4
0.3	3.3	2.6	2.2	2.1	2.0	1.7	2.1	2.0	1.5	2.3	0.3	2.4	2.1	1.7	2.0	2.3	2.8	4.2	5.3	5.9	7.0
0.4	0.3	-0.5	-0.6	-0.1	1.2	1.7	2.1	2.2	2.2	2.9	0.4	-0.6	0.0	1.8	2.3	2.9	3.4	3.0	3.2	3.8	4.4
0.5	3.8	4.9	5.2	6.2	6.8	6.8	8.1	8.8	9.5	9.7	0.5	4.8	6.1	6.8	8.9	9.7	10.3	10.4	11.4	12.2	13.7
0.6	3.0	3.8	5.4	6.3	6.7	7.6	7.6	7.8	7.6	7.9	0.6	3.8	6.3	7.4	7.5	7.9	7.8	9.8	11.3	10.7	11.7
0.7	2.8	4.2	4.6	5.4	5.8	5.4	5.3	5.8	5.4	5.7	0.7	4.2	5.3	5.4	5.7	5.7	7.1	7.1	7.2	6.9	7.0
0.8	4.1	3.6	3.2	2.9	1.6	1.7	1.0	0.8	0.9	1.3	0.8	3.9	2.8	1.2	0.8	1.3	1.6	2.6	3.0	2.7	2.9
0.9	0.2	1.2	2.5	2.8	3.5	3.8	4.7	5.2	6.3	6.1	0.9	1.2	2.8	3.8	5.2	6.0	5.7	6.2	6.2	6.5	6.3
1.0	1.5	1.6	1.6	1.7	2.3	2.2	2.2	2.1	2.1	2.1	1.0	1.6	1.7	2.2	2.1	2.1	2.3	2.3	2.3	2.5	2.5
average	2.4	2.9	3.5	3.8	4.1	4.3	4.5	4.6	4.7	5.1	average	2.9	3.8	4.2	4.5	5.2	5.5	6.3	6.7	7.2	7.8

Table 7.3 summarizes the best results of improvements above the inverse document frequency method, for each collection we obtained in the experiments. We do not have a systematic method to determine the coefficients and the learning times shown in the table.

Table 7.3 Best results			
Collection	CISI	MEDLARS	CRN4NUL
best result(%)	15.35	2.37	7.49
coefficient	0.2	0.2	0.1
learning times	4	3	1

From Fig.7.1-2, we observe the following results.

- (1) Improvement in performance is observed in every collection when the learning coefficient is small(0.002-0.016).
- (2) The improvement of MEDLARS is small compared to those of CISI and CRN4NUL.
- (3) The performance is rapidly deteriorated in CRN4NUL after the first learning with the coefficient 0.1.

One reason of (2) is that the size of learning data of MEDLARS is small compared to the other collections. In MEDLARS, 20 queries are used for learning in contrast to 61 queries of CISI and 110 queries of CRN4NUL. Another reason is that a term in  $Q_2$  does not always appear in  $Q_1$ . As described in 4.2., criterion1 can not be satisfied by MEDLARS in the partition of the query set, and only 22 query terms out of 88 total query terms contained in  $Q_2$  appear in  $Q_1$ .

The reason of (1) and (3) is that a weight of a query term of every term frequency is approaching to its optimal weight after each time of learning as far as a learning coefficient is small. On the other hand, when a learning coefficient is large, a weight of some query terms may overshoot its optimal weight and is overestimated in a query as shown in the following example.

Example Suppose that term  $T_j$  has document frequency one, and three queries use term  $T_j$  in a learning query set. Optimal weights for the queries and the initial weight are as follows(Fig.7.3(a)).

$$w_{opt}^{(1)} = 2.0$$

$$w_{opt}^{(2)} = 4.0$$

$$w_{opt}^{(3)} = 3.5$$

$$w_0 = 1.0$$

If the learning coefficient  $c$  is set 0.2, then in the first time of learning,  $w_k$  is modified as follows.

$$w_1 = 1.0 + 0.2(2.0 - 1.0) = 1.2$$

$$w_2 = 1.2 + 0.2(4.0 - 1.2) = 1.76$$

$$w_3 = 1.76 + 0.2(3.5 - 1.76) = 2.11$$

$w_3$  already exceeds  $w_{opt}^{(1)}$ . After the second time of learning  $w_6$  becomes 2.68 and  $w_9$  is 2.96 after the third time of learning(Fig.7.3(b)).□

The above example shows that a learning with large coefficient or many times of learning may cause overshootings of optimal weights for some queries. This means that an unimportant term in some queries may be overestimated and deteriorate performance. If we call the coefficient a critical coefficient when using it shows good improvement after first few learnings, but deteriorates performance after that, then the critical coefficient for CRN4NUL is around 0.1. The critical coefficients for CISI and MEDLARS are not obvious like CRN4NUL, and they may be about 2.5 for CISI and 0.6 for MEDLARS as shown in Fig.7.4. Because of the equivalence of the learning coefficient and times of learning as shown in 6.2., the concept of the critical coefficient corresponds to (coefficient) × (learning times). This means that even though a small coefficient is used in learning, it may deteriorate performance after many times of learning.

## 8. Conclusions

The parameter values used in the binary independence model and the non-binary independence model are estimated by using linear regressions and some constraints and rules in the first method. The optimal weighting functions are derived from the estimations and their performances are measured and compared to the performance of the inverse document frequency method, the cosine measure, and the statistical similarity measure. In the experiments using three different collections of documents, the estimated binary independence model shows on the average about 1.9%, 16.9%, and 10.8% better performance than the inverse document frequency method, the cosine measure, and the statistical similarity measure, respectively. The estimated non-binary independence model shows on the average about 5%, 20.3%, and 14.1% better performance than the inverse document frequency method, the cosine measure, and the statistical similarity measure, respectively.

Even though the two estimated models presented here show better performances than the three other methods, the discrepancy of the performances between the estimated models and the theoretical models is immense (Table 8.1), and there seems to be much room to improve the estimation method. One reason of this discrepancy is lack of sufficient data to make estimations accurate. Another reason is that there is much ambiguity in deciding the regression coefficients uniquely. We use some empirical rules in addition to the constraints on probability values and weighting functions to decide regression coefficients uniquely. We do not think that we choose the best possible set of rules and constraints. There might be more reasonable set of rules and constraints. For instance, the rule that weights of high term frequency should be larger than those of low term frequency may be a more reasonable constraint than the constraint B2 of section 3.2., or we should take into account a suitable model of term frequency distributions[BoSw]. Another possible reason is that we discard some data of high term frequencies in the experiments of the estimated non-binary model. Aggregating the data might be a more reasonable way to estimate the coefficients than discarding them in step 4 of 3.2.

recall	eBIM	BIM	eNBIM	NBIM
0.1	0.3670	0.4845	0.3784	0.7718
0.2	0.2776	0.4208	0.2923	0.6798
0.3	0.2075	0.3228	0.2046	0.5394
0.4	0.1639	0.2809	0.1771	0.4684
0.5	0.1477	0.2472	0.1502	0.3905
0.6	0.1265	0.2234	0.1324	0.3417
0.7	0.1093	0.1646	0.1111	0.2856
0.8	0.0937	0.1207	0.0992	0.2057
0.9	0.0762	0.1007	0.0785	0.1664
1.0	0.0471	0.0515	0.0481	0.1034

A scheme for learning optimal weights of the non-binary model is presented. The experimental results show that this scheme improves performance on the average 5.4% better than the inverse document frequency method. This result is slightly better than the estimated non-binary model. Learning with a small coefficient seems to be almost constantly improving performance with times of learnings. On the other hand, a learning with a large coefficient is deteriorating performance with times of learnings.

Validity of the schemes presented here is based on the assumption that there exists some similarity between significances of terms in a learning query set and that of the queries in a query set for evaluation. In other words, we expect that a valuable query term in a learning query set is also valuable in the other set, and non-valuable term in a learning query set is also non-valuable in the other set. This seems to be true to some extent. However, the degree of significance of the term in a query may not be the same as that in another query. That is why we obtain some improvement in the experiments, but it is not sufficiently significant. Another difficulty is that mechanical ways to determine the parameters used in the schemes are hard to find out, as in the determination of the regression coefficients in the first method and the appropriate learning coefficient in the second method.

Future directions of research are as follows:

- (1) Seek a different formula which does not show deterioration as the number of learnings increases.
- (2) Seek a way to differentiate the usage of a term by one user from that of a different user. A methodology to differentiate the usage of a term in one context from the term in another context is sketched in [Yu].

- (3) Eliminate dependencies of terms. An elimination of term dependencies is attempted in [WoZW], experimented in [WoYa], and achieves some improvement.
- (4) Incorporate normalization by document size into the schemes reported here. We compared our results mainly with the basic methods-- the inverse document frequency method and the cosine measure. Other methods might give better results than the basic methods. For instance, the method based on the inverse document frequency and improved by normalization[Crof], or the method based on the cosine measure and improved by taking into account term frequencies[Salt2] gives better performance than the basic methods. The experiments shown in this paper are not intended to show that these learning schemes achieve the best performance among the known retrieval methods. Rather it shows a more fundamental fact, that is, learning does help to improve retrieval performance. We expect better performance if normalization by document size is incorporated into the learning methods.

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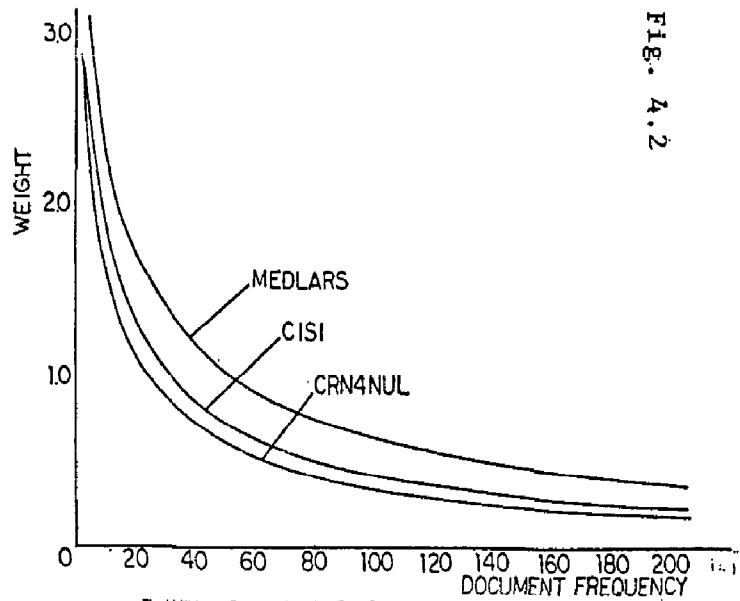


Fig. 4.2 Weighting Functions of  $d(R)$  for Three Collections

Fig. 4.2

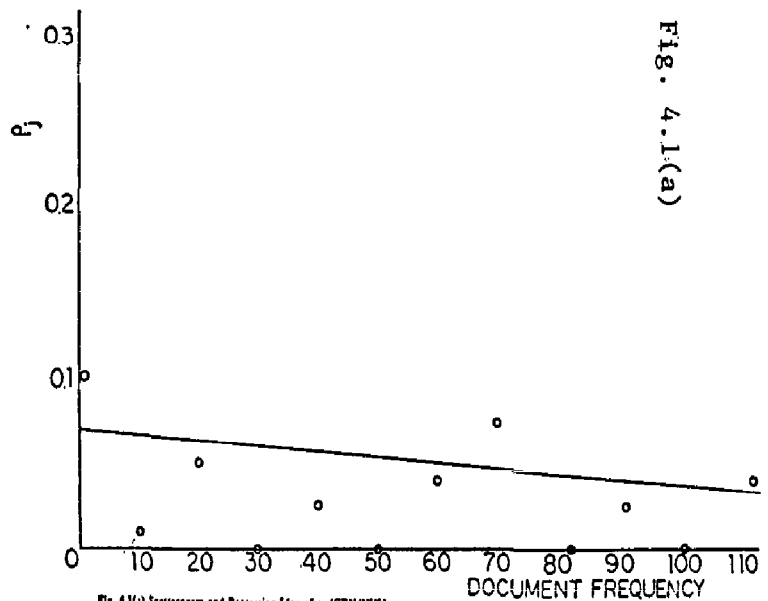


Fig. 4.1(a) Scattergram and Regression Line of  $p_j$  (CRN4NUL)

Fig. 4.1(a)

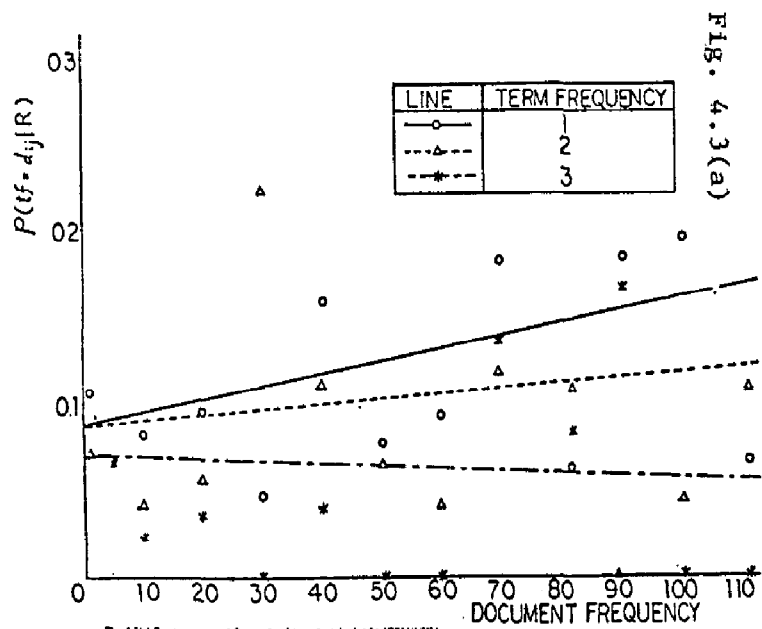


Fig. 4.3(a) Scattergram and Regression Line of  $p(tf = d_j | R)$  (CRN4NUL)

Fig. 4.3(a)

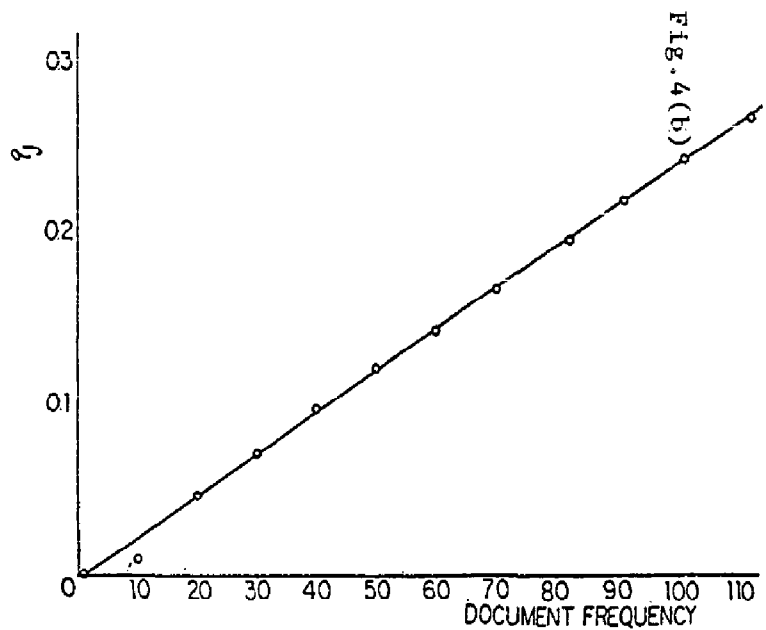


Fig. 4(b)



Fig. 4.3(b)

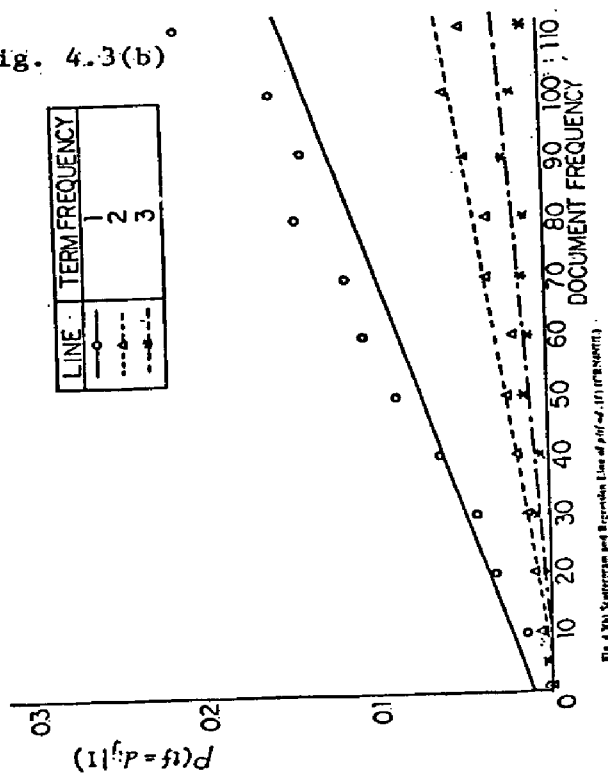


Fig. 4.3(b) Scattergram and Regression Line of  $p(f)$  vs.  $f$  (R<sup>2</sup>=0.9997)

Fig. 4.4

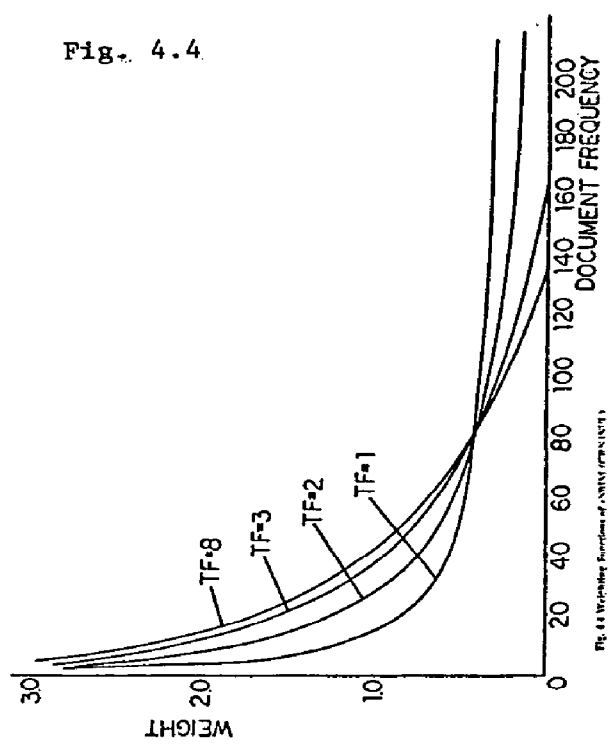


Fig. 4.4 Weibulling Functions of ARISS (EMPHASIS)

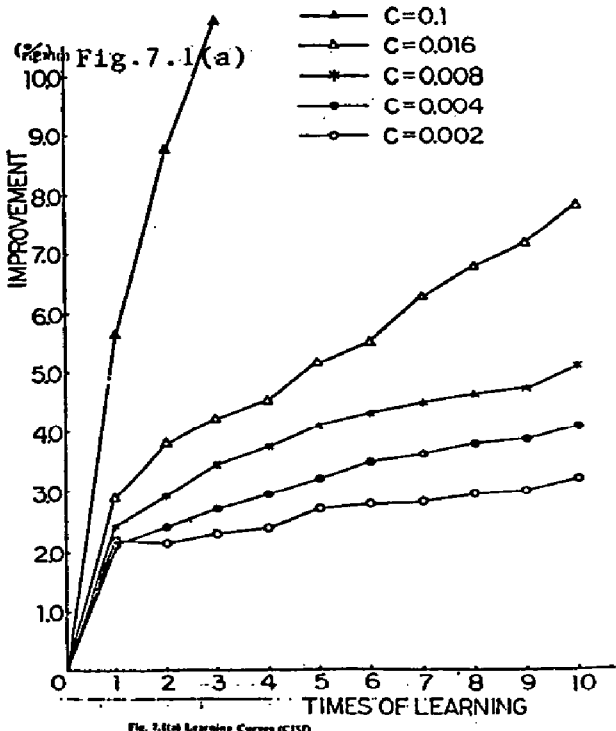


Fig. 7.1(a) Learning Curves (CISD)

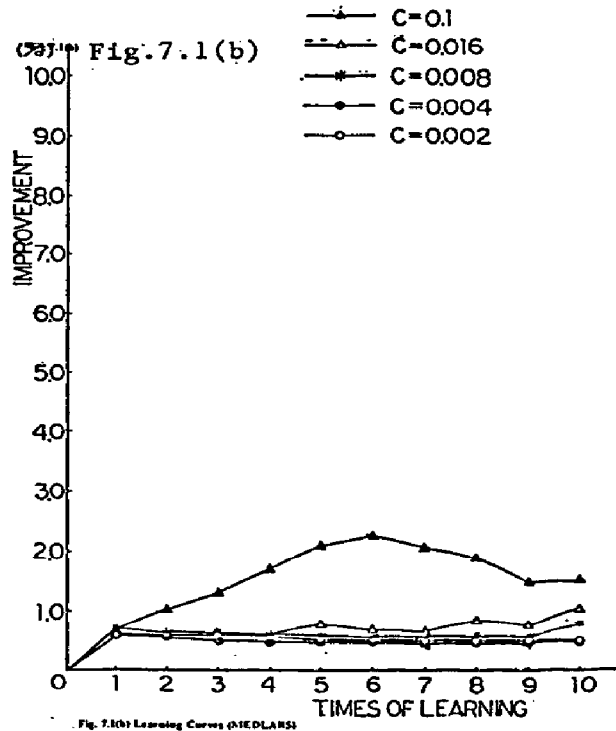


Fig. 7.1(b) Learning Curves (MEDLARS)

