Quantum Games and Minimum Entropy

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Abstract

This paper analyze Nash's equilibrium (maximum utility MU) and its relation with the order state (minimum entropy ME). I introduce the concept of minimum entropy as a paradigm of both Nash-Hayek's equilibrium. The ME concept is related to Quantum Games. One question arises after completing this exercise: What do the Quantum Mechanics postulates indicate about Game Theory and Economics?

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1 Introduction

The quantum games and the quantum computer are closely-related. The science of Quantum Computer is one of the modern paradigms in Computer Science and Game Theory [2], [3], [4], [5], [6] [7], [8]. Quantum Computer increases processing speed to have a more effective data-base search. The quantum games incorporate Quantum Theory to Game Theory [4] algorithms. This simple extrapolation allows the Prisoner's Dilemma to be solved and demonstrates that the cooperative equilibrium is viable and stable with a probability differential of zero.

In [4], [5] and [6], the analogy between Quantum Games and Game Theory is expressed as: "At the most abstract level, game theory is about numbers of entities that are efficiently acting to maximize or minimize. For a quantum physicist, it is then legitimate to ask: what happens if linear superpositions of these actions are allowed for?, that is, if games are generalized into the quantum domain. For a particular case of the Prisoner's Dilemma, we show that this game ceases to pose a dilemma if quantum strategies are implemented for." They demonstrate that classical strategies are particular quantum strategies cases.

Eisert, Wilkens, and Lewenstein, 2001, not only provide a physical model of quantum strategies but also express the idea of identifying moves using quantum operations and quantum properties. This approach appears to be fruitful in at least two ways. On one hand, several recently proposed quantum information application theories can already be conceived as competitive situations, where several factors which have opposing motives interact. These parts may apply quantum operations using a bipartite quantum system. On the other hand, generalizing decision theory in the domain of quantum probabilities seems interesting, as the roots of game theory are partly rooted in probability theory. In this context, it is of interest to investigate what solutions are attainable if superpositions of strategies are allowed [10], [14], [15] and [16]. A game is also related to the transference of information. It is possible to ask: What happens if these carriers of information are applied to be quantum systems, where Quantum information is a fundamental notion of information [8] . Nash's equilibria concept as related to quantum games is essentially the same as that of game theory, but the most important difference is that the strategies appear as a function of quantum properties in the physical system [14], [15], [19] and [20].

Econophysics is a new branch of scientific development that seeks to establish analogies between economics and physics in a particular case (analogies between Game Theory and Quantum Mechanic). The establishment of analogies is a creative way to apply the idea of cooperative equilibrium. The result of this cooperative equilibrium will allow synergies between these two sciences to occur. From my point of view, the power of physics is its ability to equilibrium formal treatment in stochastic dynamic systems. Therefore, the power of economics resides in its formal study of rationality, cooperation and the non-cooperative equilibrium.

Econophysics is the beginning of a unification stage of the systemic approach of scientific thought. I demonstrate that it is the beginning of a unification stage, but it is also related to the remaining sciences.

This paper essentially explains the relationship which exists among Quantum Mechanics, Nash's equilibria, and the Minimum Entropy Principle.

This paper is organized as follows:

Section 2: Quantum Games or Minimum Entropy, Section 3: Application of the Model. Section 4: Conclusion.

2 Quantum Games or Minimum Entropy

Let $\Gamma = (K, S, v)$ be a game with $n-$players, where K is the set of players $k = 1, ..., n$. The finite set S_k of cardinality $l_k \in N$ is the set of pure strategies of each player, where $k \in K$, $s_{kj_k} \in S_k$, $\quad j_k = 1, ..., l_k$ and $S = \Pi_K S_k$ represents the set of pure strategy profiles with $s \in S$ an element of that set, $l = l_1, l_2, ..., l_n$ represents the cardinality of S . The vectorial function $\mathsf{v} : S \to R^n$ associates every profile $s \in S$, where the vector of utilities $\mathsf{v}(s) = (v^1(s), ..., v^n(s))^T$, and $v^k(s)$ designates the utility of the player k facing the profile s. In order to understand calculus easier, we write the function $v^k(s)$ in one explicit way $v^k(s) = v^k(j_1, j_2, ..., j_n)$.The matrix $\mathsf{v}_{n,l}$ represents all points of the Cartesian product $\Pi_{k \in K} S_k$, [2],[9], [16] and [18]. The vector $\mathsf{v}^k(\mathsf{s})$ is the $k-$ column of v.

If the mixed strategies are allowed, then we have:

$$\Delta(S_k) = \left\{ \mathsf{p}^k \in R^{l_k} : \sum_{j_k=1}^{l_k} p^k_{j_k} = 1 \right\}$$

the unit simplex of the mixed strategies of player $k \in K$, and $\mathsf{p}^k = (p^k_{j_k})$ the probability vector. The set of profiles in mixed strategies is the polyhedron Δ with $\Delta = \Pi_{k \in K}\ \Delta(S_k)$, where $\mathsf{p} = (p^1_{j_1}, p^2_{j_2} ..., p^n_{j_n})$, and $\mathsf{p}^k = (p^k_1, p^k_2, ..., p^k_{l_n})^T$. Using the Kronecker product, $\otimes$ it is possible to write[1]:

$$\mathsf{p} = \mathsf{p}^1 \otimes \mathsf{p}^2 \otimes ... \otimes \mathsf{p}^{k-1} \otimes \mathsf{p}^k \otimes \mathsf{p}^{k+1} \otimes ... \otimes \mathsf{p}^n \qquad (1)$$

$$\mathsf{p}^{(-k)} = \mathsf{p}^1 \otimes \mathsf{p}^2 \otimes ... \otimes \mathsf{p}^{k-1} \otimes \mathsf{I}^k \otimes \mathsf{p}^{k+1} \otimes ... \otimes \mathsf{p}^n \qquad (1.1)$$

[1] We use bolt in order to represent vector or matrix.

where

$$\begin{aligned} \mathsf{I}^k &= (1,1,...,1)^T, \quad [\mathsf{I}^k]_{l_k,1} && (2) \\ \mathsf{o}^k &= (0,0,...,0)^T, \quad [\mathsf{o}^k]_{l_k,1} && (2.1) \end{aligned}$$

The $n-$ dimensional function $\overline{\mathsf{u}} : \Delta \to R^n$ is associated with every profile in mixed strategies and the vector of expected utilities

$$\overline{\mathsf{u}}(\mathsf{p}) = \left(\overline{u^1}(\mathsf{p}, \mathsf{v}(s)), .., \overline{u^n}(\mathsf{p}, \mathsf{v}(s))\right)^T \qquad (3)$$

where $\overline{u^k}(\mathsf{p}, \mathsf{v}(s))$ is the expected utility of the player k. Every $\overline{u^k_{j_k}} = \overline{u^k_{j_k}}(\mathsf{p}^{(-k)}, \mathsf{v}(s))$ represents the expected utility of each player's strategy and the vector u^k is noted $\mathsf{u}^k = (\overline{u^k_1}, \overline{u^k_2}, ..., \overline{u^k_n})^T$.

$$\begin{aligned} \overline{u_k} &= \sum_{j_k=1}^{l_k} \overline{u^k_{j_k}}(\mathsf{p}^{(-k)}, v(s)) p^k_{j_k} && (4) \\ \overline{\mathsf{u}} &= \mathsf{v}'\mathsf{p} && (4.1) \\ \mathsf{u}^k &= \left(\mathsf{I}^k \otimes \mathsf{v}^k\right) \mathsf{p}^{(-k)} && (4.2) \end{aligned}$$

The triplet $(K, \Delta, \overline{\mathsf{u}}(\mathsf{p}))$ designates the extension of the game Γ with the mixed strategies. We get Nash's equilibrium (the maximization of utility [2],[9],[16] and [18]) if and only if $\forall k$, p, the inequality $\overline{u^k}(\mathsf{p}^*) \geq \overline{u^k}((\mathsf{p}^k)^*, \mathsf{p}^{(-k)})$ is respected .

Another way to calculate Nash's equilibrium [9,16,18], is leveling the values of the expected prospective utilities of each strategy, when possible.

$$\begin{aligned} \overline{u^k_1}\left(\mathsf{p}^{(-k)}, v(s)\right) &= \overline{u^k_2}\left(\mathsf{p}^{(-k)}, v(s)\right) = ... = \overline{u^k_{j_k}}\left(\mathsf{p}^{(-k)}, v(s)\right) && (5) \\ \sum_{j_k=1}^{l_k} p^k_{j_k} &= 1 \qquad \forall k = 1, ..., n && (5.1) \\ \sigma^2_k &= \sum_{j_k=1}^{l_k} \left(\overline{u^k_{j_k}}\left(\mathsf{p}^{(-k)}, v(s)\right) - \overline{u}_k\right)^2 p^k_{j_k} = 0 && (5.2) \end{aligned}$$

If the resulting system of equations doesn't result in $\left(\mathsf{p}^{(-k)}\right)^*$, then we propose the Minimum Entropy Method [11] and [12]. This method is expressed as $Min_{\mathsf{p}}\left(\sum_k H_k(\mathsf{p})\right)$, where $\sigma^2_k(\mathsf{p}^*)$ is the standard deviation and $H_k(\mathsf{p}^*)$ is the entropy of each player k.

$$\begin{aligned} \sigma^2_k(\mathsf{p}^*) &\leq \sigma^2_k\left(\left(\mathsf{p}^k\right)^*, \mathsf{p}^{(-k)}\right) \text{ or} && (6) \\ H_k(\mathsf{p}^*) &\leq H_k\left(\left(\mathsf{p}^k\right)^*, \mathsf{p}^{(-k)}\right) && (6.1) \end{aligned}$$

2.1 Minimum Entropy Method

Theorem 1 (Minimum Entropy Theorem). The game entropy is minimum only in mixed strategy Nash's equilibrium [11] and [12]. The entropy minimization program $Min_{\mathsf{p}}\left(\sum_k H_k(\mathsf{p})\right)$, is equal to the standard deviation minimization program $Min_{\mathsf{p}}\left(\Pi_k \sigma_k(\mathsf{p})\right)$, when $\left(\overline{u^k_{j_k}}\right)$ has gaussian density function or multinomial logit.

According to Hayek, equilibrium refers to the order state or minimum entropy. The order state is opposite to entropy (disorder measure). There are some intellectual influences and historical events which inspired Hayek to develop the idea of a spontaneous order. Here, we present the technical tools needed in order to study the order state [11], [12], [17] and [17].

Case 1 If the probability density of a variable X is normal: $N(\mu_k, \sigma_k)$, then its entropy is minimum for the minimum standard deviation $(H_k)_{\min} \Leftrightarrow (\sigma_k)_{\min}$. $\forall k = 1, ..., n$.

Proof. Let the entropy function $H_k = -\int_{-\infty}^{+\infty} p(x)\ln(p(x))dx$ and $p(x)$ the normal density function.

$$H_k = -\int_{-\infty}^{+\infty} p(x)\ln(p(x))dx \tag{7}$$

$$H_k = -\int_{-\infty}^{+\infty} p(x)\left(\ln\left(\frac{1}{\sqrt{2\pi\sigma_k^2}}\right) - \left(\frac{x-\overline{u_k}}{\sqrt{2}\sigma_k}\right)^2\right)dx \tag{7.1}$$

developing the integral we have

$$H_k = \ln\left(\sqrt{2\pi}\sigma_k\right)\int_{-\infty}^{+\infty} p(x)dx \tag{8}$$

$$+\frac{1}{2\sigma_k^2}\int_{-\infty}^{+\infty}(x-\overline{u_k})^2 p(x)dx \tag{8.1}$$

$$= \ln\left(\sqrt{2\pi}\sigma_k\right) + \frac{\sigma_k^2}{2\sigma_k^2} = \left(\frac{1}{2} + \ln\sqrt{2\pi}\right) + \ln\sigma_k \tag{8.2}$$

For a game to $n-$ players, the total entropy can be written as follows:

$$\sum_{k=1}^{n} H_k = n\left(\frac{1}{2} + \ln\sqrt{2\pi}\right) + \ln(\Pi_{k=1}^{n}\ \sigma_k) \tag{9}$$

$$Min_{\mathsf{p}}\left(\sum_k H_k(\mathsf{p})\right) \Leftrightarrow Min_{\mathsf{p}}\left(\Pi_k\sigma_k(\mathsf{p})\right) \tag{9.1}$$

after making a few more calculations, it is possible to demonstrate that

$$Min_{\mathsf{p}}\left(\sum_{k=1}^{n}\sigma_k(\mathsf{p})\right) \Rightarrow Min_{\mathsf{p}}\left(\Pi_k\sigma_k(\mathsf{p})\right) \tag{10}$$

■

The entropy or measure of the disorder is directly proportional to the standard deviation or measure of uncertainty. Clausius, who discovered the entropy idea, presents it as both an evolutionary measure and as the characterization of reversible and irreversible processes [1], [3], [7] and [13].

Case 2 If the probability function of $\overline{u_{j_k}^k}(\mathsf{p}^{(-k)}, v(s))$ is a multinomial logit of parameter λ, then its entropy is minimum and its standard deviation is minimum for $\lambda \rightarrow \infty$.

Proof. Let $p^k_{j_k}$ the probability for $k \in K$, $j_k = 1, ..., l_k$

$$p^k_{j_k} = \frac{e^{\lambda u^k_{j_k}(\mathsf{p}^{-k})}}{\sum_{j_k=1}^{l_k} e^{\lambda u^k_{j_k}(\mathsf{p}^{-k})}} = \frac{e^{\lambda u^k_{j_k}(\mathsf{p}^{-k})}}{Z_k} \tag{11}$$

where $Z_k\left(\lambda, \mathsf{u}^k(\mathsf{p}^{-k})\right) = \sum_{j_k=1}^{l_k} e^{\lambda u^k_{j_k}(\mathsf{p}^{-k})}$ represents the partition utility function [1], [11] and [12].

The entropy $H_k(\mathsf{p}^k)$, expected utility $E\{u^k_{j_k}(\mathsf{p}^{-k})\} = \overline{u^k}(p)$, and variance $Var\{u^k_{j_k}(\mathsf{p}^{-k})\}$ will be different for each player k.

$$H_k(\mathsf{p}^k) = -\sum_{j_k=1}^{l_k} p^k_{j_k} \ln(p^k_{j_k}) \tag{12}$$

$$\overline{u^k}(p) = \sum_{j_k=1}^{l_k} p^k_{j_k} u^k_{j_k} \tag{12.1}$$

$$Var\left\{u^k_{j_k}(\mathsf{p}^{-k})\right\} = \sum_{j_k=1}^{l_k} p^k_{j_k}(u^k_{j_k} - \overline{u^k})^2 \tag{12.2}$$

Using the explicit form of $p^k_{j_k}$, we obtain the entropy, the expected utility and the variance [11] and [12]:

$$H_k(\mathsf{p}^k) = \ln(Z_k) - \lambda\overline{u^k}(p) \tag{13}$$

$$\overline{u^k}(p) = \frac{\partial \ln(Z_k)}{\partial \lambda} \tag{13.1}$$

$$\frac{\partial \overline{u^k}(p)}{\partial \lambda} = Var\{u^k_{j_k}(\mathsf{p}^{-k})\} = \sigma_k^2 \tag{13.2}$$

The equation $\frac{\partial H_k(\mathsf{p}^k)}{\partial \lambda} = -\lambda \sigma_k^2$ can be obtained using the last seven equations; it explains that, when entropy diminishes, rationality increases.

The rationality increases from an initial value of zero when the entropy arrives at its maximum value, and drops to its minimum value when the rationality spreads toward the infinite value: $Lim_{\lambda \to \infty} H_k(\mathsf{p}^k(\lambda)) = \min(H_k)$

The standard deviation is minimum in Nash's equilibria [2] and [16].

If rationality increases, then Nash's equilibria can be reached when the rationality spreads to its infinite value: $Lim_{\lambda \to \infty} \sigma_k(\lambda) = 0$.

Using the logical chain that has just been demonstrated, we can conclude that the entropy diminishes when the standard deviation diminishes:

$$(H_k(\mathsf{p}) = (H_k(\mathsf{p}))_{\mathsf{min}}) \Leftrightarrow (\sigma_k(\lambda) = (\sigma_k)_{\mathsf{min}}) \tag{14}$$

$$Min_{\mathsf{p}}\left(\sum_k H_k(\mathsf{p})\right) \Leftrightarrow Min_{\mathsf{p}}\left(\Pi_k \sigma_k(\mathsf{p})\right) \tag{14.1}$$

after making a few more calculations, it is possible to demonstrate that

$$Min_{\mathsf{p}}\left(\sum_{k=1}^{n} \sigma_k(\mathsf{p})\right) \Rightarrow Min_{\mathsf{p}}\left(\Pi_k \sigma_k(\mathsf{p})\right) \tag{15}$$

■

Remark 1 The entropy H_k for gaussian probability density and multinomial logit is written as $\left(\frac{1}{2}+\ln\sqrt{2\pi}\right)+\ln\sigma_k$ and $\ln(\sum_{j_k=1}^{l_k} e^{\lambda\overline{u_{j_k}^k}})-\lambda\overline{u^k}$

Case 3 The special case of Minimum Entropy is when $\sigma_k^2=0$ and the utility function value of each strategy $\overline{u_{j_k}^k}(\mathsf{p}^{(-k)},v(s))=\overline{u^k}$, is the same $\forall j_k,\forall k$.

3 Application of the Model

Let $\Gamma=(K,S,v)$ be a game to 3−players, with K the set of players $k=1,2,3$. The finite set S_k of cardinality $l_k\in N$ is the set of pure strategies of each player where $k\in K$, $s_{kj_k}\in S_k$, $j_k=1,2,3$ and $S=\Pi_k S_k$ represents the set of pure strategy profiles which have $s\in S$ as an element of that set, and $l=3*3*3=27$ represents the cardinality of S. The vectorial function $\mathsf{v}:S\to R^3$ associates every profile $s\in S$ with the vector of utilities $\mathsf{v}(s)=(v^1(s),...,v^3(s))$, where $v^k(s)$ designates the utility of the player k using the profile s. In order to use calculus easier, we write the function $v^k(s)$ in one explicit way: $v^k(s)=v^k(j_1,j_2,...,j_n)$.The matrix $\mathsf{v}_{3,27}$ represents all points of the Cartesian product $\Pi_K S_k$ see in (Table 1). The vector $\mathsf{v}^k(\mathsf{s})$ is the k- column of v. The graphic representation of the 3-players game (Figure 1).

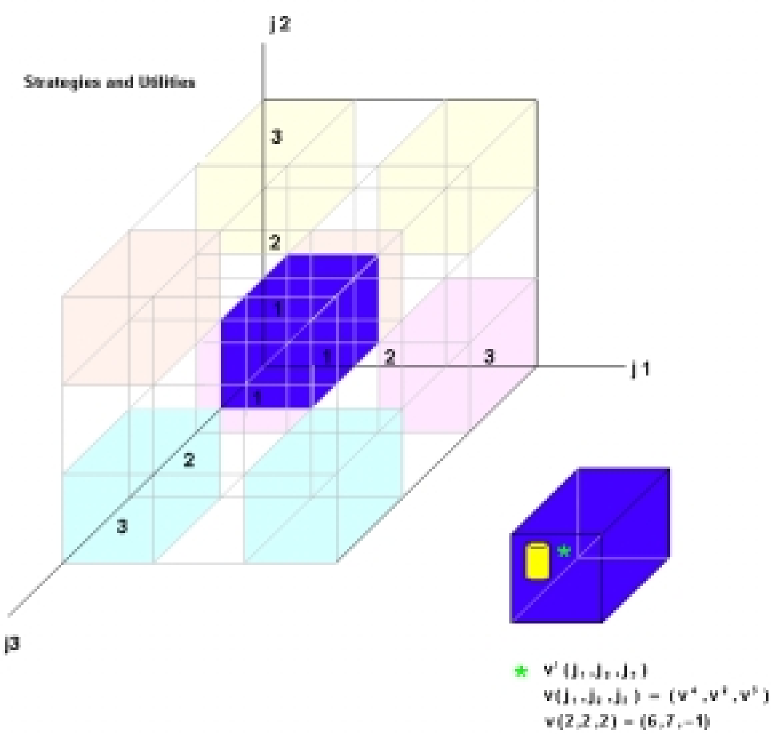

Figure 1 3-players game strategies

Table 1: Minimum Entropy: The Game of Stone-Paper-Scissors.

$$\min_{\mathsf{p}}(H_1 + H_2 + H_3) \Leftrightarrow Min_{\mathsf{p}}\,(\sigma_1(\mathsf{p})\sigma_2(\mathsf{p})\sigma_3(\mathsf{p}))$$

Nash Utilities and Standard Deviations

$Min_p\ (\sigma^1\sigma^2\sigma^3) = 0$

u^1	u^2	u^3
0.5500	0.5500	0.5500
σ^1	σ^2	σ^3
0.0000	0.0000	0.0000
H_1	H_2	H_3
0.0353	0.0353	0.0353

Nash Equilibria: Probabilities, Utilities

Player 1			Player 2			Player 3		
p^1_1	p^1_2	p^1_3	p^2_1	p^2_2	p^2_3	p^3_1	p^3_2	p^3_3
0.3333	0.3333	0.3333	0.3333	0.3333	0.3333	0.3300	0.3300	0.3300
u^1_1	u^1_2	u^1_3	u^2_1	u^2_2	u^2_3	u^3_1	u^3_2	u^3_3
0.5500	0.5500	0.5500	0.5500	0.5500	0.5500	0.5556	0.5556	0.5556
$p^1_1\,u^1_1$	$p^1_2\,u^1_2$	$p^1_3\,u^1_3$	$p^2_1\,u^2_1$	$p^2_2\,u^2_2$	$p^2_3\,u^2_3$	$p^3_1\,u^3_1$	$p^3_2\,u^3_2$	$p^3_3\,u^3_3$
0.1833	0.1833	0.1833	0.1833	0.1833	0.1833	0.1833	0.1833	0.1833

Kroneker Products

j_1	j_2	j_3	$v^1(j_1,j_2,j_3)$	$v^2(j_1,j_2,j_3)$	$v^3(j_1,j_2,j_3)$	p^1_{j1}	p^2_{j2}	p^3_{j3}	$p^1_{j1}p^2_{j2}$	$p^1_{j1}p^3_{j3}$	$p^2_{j2}p^3_{j3}$	$u^1(j_1,j_2,j_3)$	$u^2(j_1,j_2,j_3)$	$u^3(j_1,j_2,j_3)$
stone	stone	stone	0	0	0	0.3333	0.3333	0.3300	0.1111	0.1100	0.1100	0.0000	0.0000	0.0000
stone	stone	paper	0	0	1	0.3333	0.3333	0.3300	0.1111	0.1100	0.1100	0.0000	0.0000	0.1111
stone	stone	sccisor	1	1	0	0.3333	0.3333	0.3300	0.1111	0.1100	0.1100	0.1100	0.1100	0.0000
stone	paper	stone	0	1	0	0.3333	0.3333	0.3300	0.1111	0.1100	0.1100	0.0000	0.1100	0.0000
stone	paper	paper	0	1	1	0.3333	0.3333	0.3300	0.1111	0.1100	0.1100	0.0000	0.1100	0.1111
stone	paper	sccisor	1	1	1	0.3333	0.3333	0.3300	0.1111	0.1100	0.1100	0.1100	0.1100	0.1111
stone	sccisor	stone	1	0	1	0.3333	0.3333	0.3300	0.1111	0.1100	0.1100	0.1100	0.0000	0.1111
stone	sccisor	paper	1	1	1	0.3333	0.3333	0.3300	0.1111	0.1100	0.1100	0.1100	0.1100	0.1111
stone	sccisor	sccisor	1	0	0	0.3333	0.3333	0.3300	0.1111	0.1100	0.1100	0.1100	0.0000	0.0000
paper	stone	stone	1	0	0	0.3333	0.3333	0.3300	0.1111	0.1100	0.1100	0.1100	0.0000	0.0000
paper	stone	paper	1	0	1	0.3333	0.3333	0.3300	0.1111	0.1100	0.1100	0.1100	0.0000	0.1111
paper	stone	sccisor	1	1	1	0.3333	0.3333	0.3300	0.1111	0.1100	0.1100	0.1100	0.1100	0.1111
paper	paper	stone	1	1	0	0.3333	0.3333	0.3300	0.1111	0.1100	0.1100	0.1100	0.1100	0.0000
paper	paper	paper	0	0	0	0.3333	0.3333	0.3300	0.1111	0.1100	0.1100	0.0000	0.0000	0.0000
paper	paper	sccisor	0	0	1	0.3333	0.3333	0.3300	0.1111	0.1100	0.1100	0.0000	0.0000	0.1111
paper	sccisor	stone	1	1	1	0.3333	0.3333	0.3300	0.1111	0.1100	0.1100	0.1100	0.1100	0.1111
paper	sccisor	paper	0	1	0	0.3333	0.3333	0.3300	0.1111	0.1100	0.1100	0.0000	0.1100	0.0000
paper	sccisor	sccisor	0	1	1	0.3333	0.3333	0.3300	0.1111	0.1100	0.1100	0.0000	0.1100	0.1111
sccisor	stone	stone	0	1	1	0.3333	0.3333	0.3300	0.1111	0.1100	0.1100	0.0000	0.1100	0.1111
sccisor	stone	paper	1	1	1	0.3333	0.3333	0.3300	0.1111	0.1100	0.1100	0.1100	0.1100	0.1111
sccisor	stone	sccisor	0	1	0	0.3333	0.3333	0.3300	0.1111	0.1100	0.1100	0.0000	0.1100	0.0000
sccisor	paper	stone	1	1	1	0.3333	0.3333	0.3300	0.1111	0.1100	0.1100	0.1100	0.1100	0.1111
sccisor	paper	paper	1	0	0	0.3333	0.3333	0.3300	0.1111	0.1100	0.1100	0.1100	0.0000	0.0000
sccisor	paper	sccisor	1	0	1	0.3333	0.3333	0.3300	0.1111	0.1100	0.1100	0.1100	0.0000	0.1111
sccisor	sccisor	stone	0	0	1	0.3333	0.3333	0.3300	0.1111	0.1100	0.1100	0.0000	0.0000	0.1111
sccisor	sccisor	paper	1	1	0	0.3333	0.3333	0.3300	0.1111	0.1100	0.1100	0.1100	0.1100	0.0000
sccisor	sccisor	sccisor	0	0	0	0.3333	0.3333	0.3300	0.1111	0.1100	0.1100	0.0000	0.0000	0.0000

4 Conclusion

1 An immediate application of quantum games in the science of economics is related to the principal-agent relationship. Specifically, we can use m to represent types of agents in the adverse selection model. In moral risk, quantum games could be used as a discrete or continuous set of efforts.

2 In this paper we have demonstrated that the Nash-Hayek equilibrium opens new doors so that entropy in game theory can be used. Remembering that the primary way to prove Nash's equilibria is through utility maximization, we can affirm that human behavior arbitrates between these two stochastic-utility (benefits) $U(p(\mathsf{x}))$ and entropy (risk or disorder) $H(p(\mathsf{x}))$ elements. Accepting that the stochastic-utility/entropy $\left[\frac{U(p(\mathsf{x}))}{H(p(\mathsf{x}))}\right]$ relationship is equivalent to the well-known benefits/cost ratio, we present a new way to calculate equilibria: $Max_{\mathsf{x}}\left(\frac{U(p(\mathsf{x}))}{H(p(\mathsf{x}))}\right)$, where $p(x)$ probability function and $\mathsf{x} = (x_1, x_2, ..., x_n)$ exogenous variables.

3 This paper, which uses Kronecker product $\otimes$, represents an easy, new formalization of game $(K, \Delta, \overline{\mathsf{u}}(\mathsf{p}))$,which extends the game Γ to the mixed strategies.

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