Introduction to Ensemble Learning

Featuring Successes in the Netflix Prize Competition

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Outline

- Introduction
  - Bias and variance problems
- The Netflix Prize
  - Success of ensemble methods in the Netflix Prize
- Why Ensemble Methods Work
- Algorithms
  - AdaBoost
  - BrownBoost
  - Random forests

1-Slide Intro to Supervised Learning

We want to approximate a function,

$$f = f(\mathbf{x})$$

Given examples,

$$D = \{y_i, x_{i1}, x_{i2}, \cdots, x_{in}\}_1^N$$

Find a function *h* among a fixed subclass of functions for which the error *E*(*h*) is minimal,

$$E\left\{(t_i - y_i)^2\right\} = \underbrace{Var\{noise\}}_{\text{Independent of } h} + \underbrace{bias^2}_{\text{The distance from of } f} + \underbrace{Var\{y_i\}}_{\text{Variance of the predictions}}$$

Bias and Variance

<u>Bias Problem</u>

- The hypothesis space made available by a particular classification method does not include sufficient hypotheses

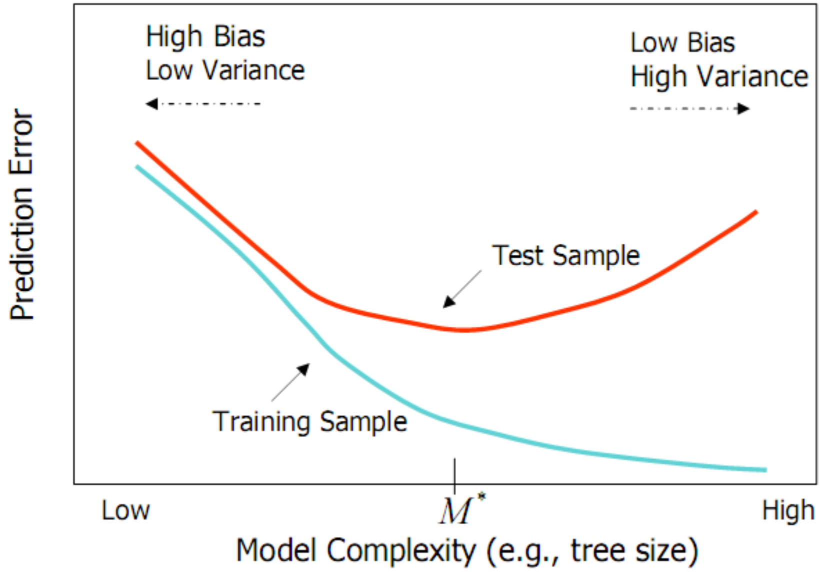
<u>Variance Problem</u>

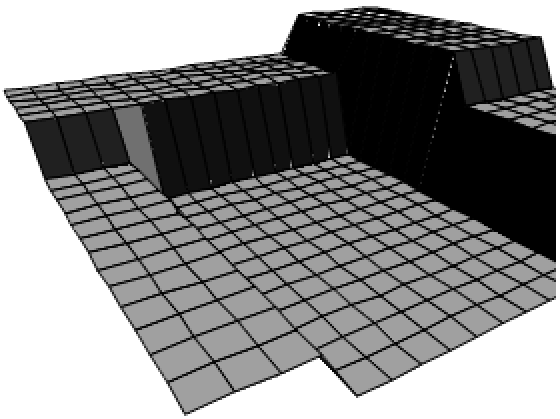
- The hypothesis space made available is too large for the training data, and the selected hypothesis may not be accurate on unseen data

Bias and Variance

Decision Trees

- Small trees have high bias.
- Large trees have high variance. Why?





Decision Tree

from Elder, John. From Trees to Forests and Rule Sets - A Unified Overview of Ensemble Methods. 2007.

Definition

Ensemble Classification

Aggregation of predictions of multiple classifiers with the goal of improving accuracy.

Teaser: How good are ensemble methods?

Let’s look at the Netflix Prize Competition…

Netflix Prize *Began October 2006*

- Supervised learning task
  - Training data is a set of users and ratings (1,2,3,4,5 stars) those users have given to movies.
  - Construct a classifier that given a user and an unrated movie, correctly classifies that movie as either 1, 2, 3, 4, or 5 stars

- $1 million prize for a 10% improvement over Netflix's current movie recommender/classifier (MSE = 0.9514)

Just three weeks after it began, at least 40 teams had bested the Netflix classifier.

Top teams showed about 5% improvement.

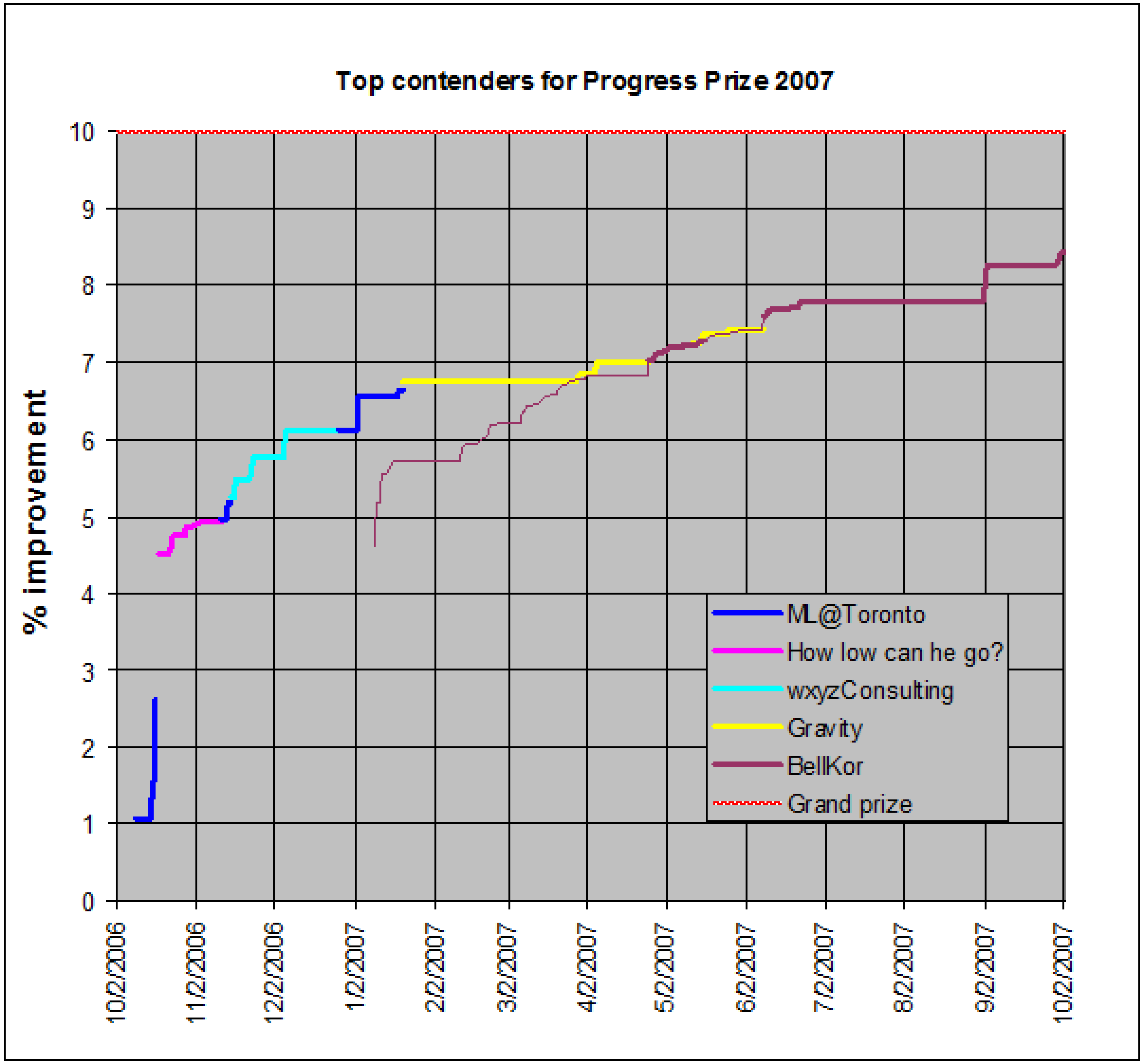
Netflix Prize

me | Rules | Leaderboard | Register | Update | Submit | Download

Leaderboard

Team Name	Best Score	% Improvement
No Grand Prize candidates yet	--	--
Grand Prize - RMSE <= 0.8563		
How low can he go?	0.9046	4.92
ML@UToronto A	0.9046	4.92
ssorkin	0.9089	4.47
wxyzconsulting.com	0.9103	4.32
The Thought Gang	0.9113	4.21
NIPS Reject	0.9118	4.16
simonfunk	0.9145	3.88
Bozo_The_Clown	0.9177	3.54
Elliptic Chaos	0.9179	3.52
datcracker	0.9183	3.48
Foreseer	0.9214	3.15
bsdfish	0.9229	3.00
Three Blind Mice	0.9234	2.94
Bocsimacko	0.9238	2.90
Remco	0.9252	2.75
karmatics	0.9301	2.24
Chapelator	0.9314	2.10
Flmod	0.9325	1.99
mthrox	0.9328	1.96

However, improvement slowed…



from http://www.research.att.com/~volinsky/netflix/

Today, the top team has posted a 8.5% improvement.

Ensemble methods are the best performers…

--	No Progress Prize candidates yet	--	--
Progress Prize - RMSE <= 0.8625			
1	BellKor	0.8705	8.50
Progress Prize 2007 - RMSE = 0.8712 - Winning Team: KorBell			
2	KorBell	0.8712	8.43
3	When Gravity and Dinosaurs Unite	0.8717	8.38
4	Gravity	0.8743	8.10
5	basho	0.8746	8.07
6	Dinosaur Planet	0.8753	8.00
7	ML@UToronto A	0.8787	7.64
8	Arek Paterek	0.8789	7.62
9	NIPS Reject	0.8808	7.42
10	Just a guy in a garage	0.8834	7.15
11	Ensemble Experts	0.8841	7.07
12	mathematical capital	0.8844	7.04
13	HowLowCanHeGo2	0.8847	7.01
14	The Thought Gang	0.8849	6.99
15	Reel Ingenuity	0.8855	6.93
16	strudeltamale	0.8859	6.88
17	NIPS Submission	0.8861	6.86
18	Three Blind Mice	0.8869	6.78
19	TrainOnTest	0.8869	6.78
20	Geoff Dean	0.8869	6.78
21	Rookies	0.8872	6.75
22	Paul Harrison	0.8872	6.75
23	ATTEAM	0.8873	6.74
24	wxyzconsulting.com	0.8874	6.73
25	ICMLsubmission	0.8875	6.72
26	Efratko	0.8877	6.70
27	Kitty	0.8881	6.65
28	SecondaryResults	0.8884	6.62
29	Birgit Kraft	0.8885	6.61

Rookies

"Thanks to Paul Harrison's collaboration, a simple mix of our solutions improved our result from 6.31 to 6.75"

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Arek Paterek

"My approach is to **combine the results of many methods** (also two-way interactions between them) using linear regression on the test set. The best method in my ensemble is regularized SVD with biases, post processed with kernel ridge regression"

http://rainbow.mimuw.edu.pl/~ap/ap_kdd.pdf

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U of Toronto

“When the predictions of **multiple** RBM models and **multiple** SVD models are linearly combined, we achieve an error rate that is well over 6% better than the score of Netflix’s own system.”

http://www.cs.toronto.edu/~rsalakhu/papers/rbmcf.pdf

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Gravity

Table 5: Best results of single approaches and their combinations

Method/Combination	RMSE
MF	0.9190
NB	0.9313
CL	0.9606
NB + CL	0.9275
MF + CL	0.9137
MF + NB	0.9089
MF + NB + CL	0.9089

home.mit.bme.hu/~gtakacs/download/**gravity**.pdf

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When Gravity and Dinosaurs Unite

“Our common team blends the result of team Gravity and team Dinosaur Planet.”

Might have guessed from the name…

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BellKor / KorBell

And, yes, the top team which is from AT&T…

“Our final solution (RMSE=0.8712) consists of blending 107 individual results. “

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Some Intuitions on Why Ensemble Methods Work…

Intuitions

- Utility of combining diverse, independent opinions in human decision-making
  - Protective Mechanism (e.g. stock portfolio diversity)
- **Violation of Ockham's Razor**
  - Identifying the best model requires identifying the proper "model complexity"

See Domingos, P. Occam's two razors: the sharp and the blunt. KDD. 1998.

Intuitions

Majority vote

Suppose we have 5 completely independent classifiers…

- If accuracy is 70% for each
  - 10 (.7^3)(.3^2)+5(.7^4)(.3)+(.7^5)
  - **83.7% majority vote accuracy**
- 101 such classifiers
  - **99.9% majority vote accuracy**

Strategies

<u>Boosting</u>

- Make examples currently misclassified more important (or less, in some cases)

<u>Bagging</u>

- Use different samples or attributes of the examples to generate diverse classifiers

Boosting

Make examples currently misclassified more important (or less, if lots of noise). Then combine the hypotheses given…

$$H(\mathbf{x}_i) = \sum_k w_k h_k(\mathbf{x}_i)$$

<u>Types</u>

- AdaBoost
- BrownBoost

AdaBoost Algorithm

1. Initialize Weights

$$d_1(\mathbf{x}_i) = 1/m$$

2. Construct a classifier. Compute the error.

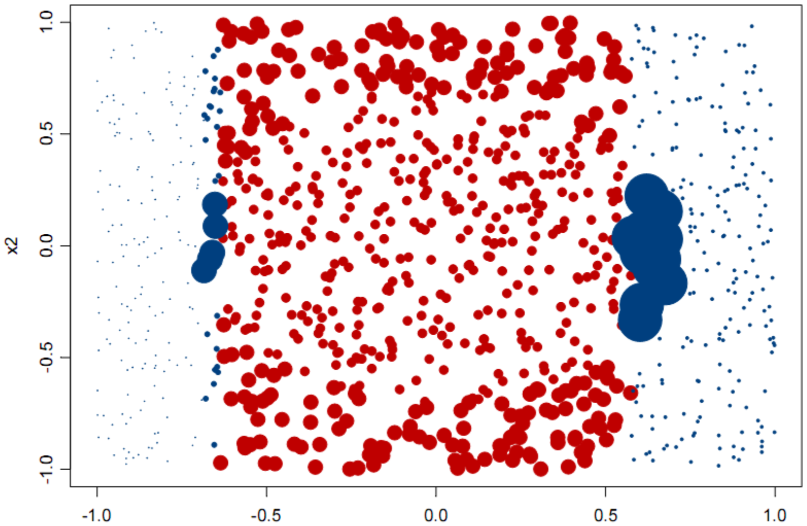
$$r = \sum_i d(\mathbf{x}_i) y_i h_k(\mathbf{x}_i)$$

3. Update the weights, and repeat step 2. …

$$d_{k+1}(\mathbf{x}_i) = d_k(\mathbf{x}_i)\frac{\exp(-w_k y_i h_k(\mathbf{x}_i))}{Z_k}$$

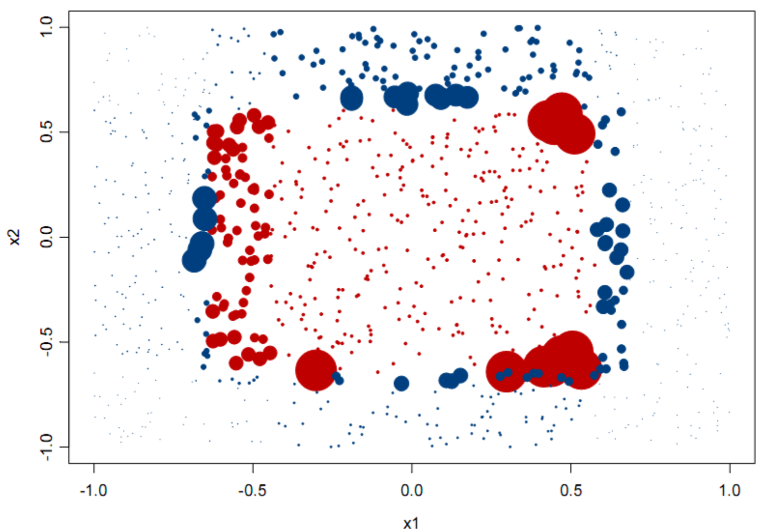
4. Finally, sum hypotheses…

$$w_k = \frac{1}{2}\ln\left(\frac{1+r}{1-r}\right)$$

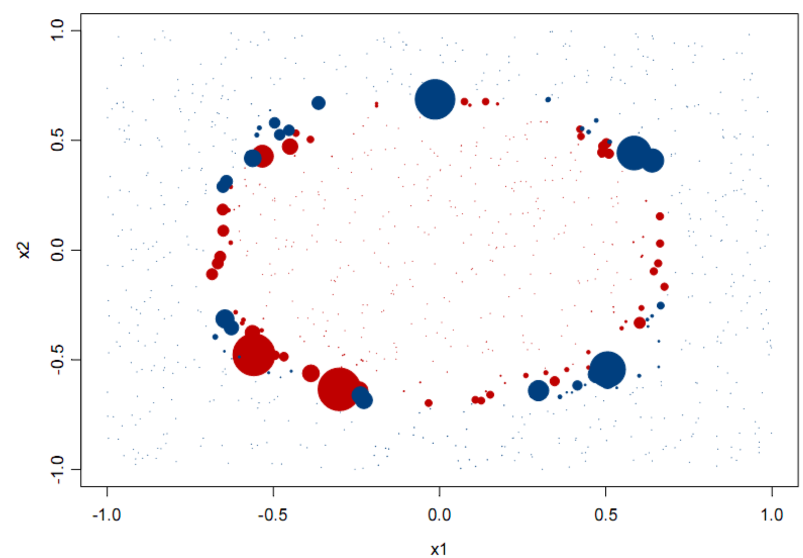


Classifications (colors) and Weights (size) after *1 iteration* Of AdaBoost

3 iterations



20 iterations



from Elder, John. From Trees to Forests and Rule Sets - A Unified Overview of Ensemble Methods. 2007.

AdaBoost

- Advantages
  - Very little code
  - Reduces variance
- Disadvantages
  - Sensitive to noise and outliers. Why?

BrownBoost

- Reduce the weight given to misclassified example

- Good (only) for very noisy data.

Bagging (Constructing for Diversity)

1. Use random samples of the examples to construct the classifiers
2. Use random attribute sets to construct the classifiers
    - Random Decision Forests

Leo Breiman

Random forests

- At every level, choose a random subset of the attributes (not examples) and choose the best split among those attributes
- Doesn’t overfit

Random forests

- Let the number of training cases be *M*, and the number of variables in the classifier be *N*.

For each tree,

1. Choose a training set by choosing *N* times with replacement from all *N* available training cases.
2. For each node, randomly choose *n* variables on which to base the decision at that node.

$$n_s = \lfloor \log_2(n) + 1 \rfloor$$

Calculate the best split based on these.

Data set	Adaboost	Selection	Forest-RI single input	One tree
Glass	22.0	20.6	21.2	36.9
Breast cancer	3.2	2.9	2.7	6.3
Diabetes	26.6	24.2	24.3	33.1
Sonar	15.6	15.9	18.0	31.7
Vowel	4.1	3.4	3.3	30.4
Ionosphere	6.4	7.1	7.5	12.7
Vehicle	23.2	25.8	26.4	33.1
German credit	23.5	24.4	26.2	33.3
Image	1.6	2.1	2.7	6.4
Ecoli	14.8	12.8	13.0	24.5
Votes	4.8	4.1	4.6	7.4
Liver	30.7	25.1	24.7	40.6
Letters	3.4	3.5	4.7	19.8
Sat-images	8.8	8.6	10.5	17.2
Zip-code	6.2	6.3	7.8	20.6
Waveform	17.8	17.2	17.3	34.0
Twonorm	4.9	3.9	3.9	24.7
Threenorm	18.8	17.5	17.5	38.4
Ringnorm	6.9	4.9	4.9	25.7

Breiman, Leo (2001). "Random Forests". Machine Learning 45 (1), 5-32

Questions / Comments?

Sources

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