Particle Swarm Optimization Approach for Multi-Objective Composite Box-Beam Design

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Abstract:-

This paper presents a multi-agent search technique to design an optimal composite box-beam helicopter rotor blade. The search technique is called particle swarm optimization ('inspired by the choreography of a bird flock'). The continuous geometry parameters (cross-sectional dimensions) and discrete ply angles of the box-beams are considered as design variables. The objective of the design problem is to achieve a) specified stiffness value and b) maximum elastic coupling. The presence of maximum elastic coupling in the composite box-beam increases the aeroelastic stability of the helicopter rotor blade. The multi-objective design problem is formulated as a combinatorial optimization problem and solved collectively using particle swarm optimization technique. The optimal geometry and ply angles are obtained for a composite box-beam design with ply angle discretizations of 10°, 15° and 45°. The performance and computational efficiency of the proposed particle swarm optimization approach is compared with various genetic algorithm based design approaches. The simulation results clearly show that the particle swarm optimization algorithm provides better solutions in terms of performance and computational time than the genetic algorithm based approaches.

Keywords:- Composite laminates, box-beam design, helicopter rotor blades, multi-objective optimization, particle swarm optimization and genetic algorithm

1 Introduction

Over two decades, the aircraft industries preferred composite structures over metallic structures because of their high strength-to-weight ratio and high specific stiffness. Another advantage of using a composite structure is that the structure can be tailored by selecting appropriate fiber materials and ply orientations to meet the specific design requirements. The composite laminate structures offer many opportunities for designers to optimize the structure for a specific or even multiple design criterions. A typical composite structural design problem has large number of discrete design variables such as number of layers, ply orientation, thickness and type of material. The flexibility in selecting these variables to meet the design requirements introduces a complexity in the design problem. In most of the design problems, certain specifications are known *a priori*, like, laminate thickness, choices for ply orientations and type of the material. Hence, the design of a composite structure reduces to search for appropriate discrete ply orientations from a given set of ply orientations and geometry parameters from a given range to achieve specified strength and stiffness. The presence of discrete and continuous design variables increases the complexity of the design problem.

In earlier studies, mathematical optimization methods were used to design composite structures. In [1, 2], the design variables are considered as continuous variables and the optimization problem was solved using conventional optimization algorithms. The optimal ply angles obtained from these continuous optimization methods were rounded off to appropriate integer values that resulted in non-optimal designs. The nonlinear and combinatorial nature of the composite structural design problem induces difficulty in using deterministic approaches for solutions. Recently, an improved hit and run global search algorithm was used to find the discrete ply angles and continuous geometry variables [3]. In hit and run global search algorithm, the computational cost to obtain global minimum solution increases exponentially with increase in number of discrete variables. Hence, designing a better search technique for composite structure is a challenging problem.

In recent years, evolutionary algorithms have been used to solve nonlinear combinatorial optimization problems. Genetic algorithms (GA), the most popular evolutionary algorithm, mimic the mechanics of natural genetics for artificial systems based on operation that are the counterparts of natural ones. In the last decade, different GA based approaches for ply orientations or stacking sequence optimization for different functional purposes have been devised and reported in literature [4–11]. A binary representation is used to code the integer design variables in genetic algorithm framework. In the binary string representation, the length of the chromosome increases with increase in the number of design variables, that affects the efficiency and convergence of the GA. An alternative to the binary string representation is the real value representation of design variables in the GA. GA's using real value or floating point representation for solutions are called real-coded genetic algorithm (RCGA) [21]. A good representation scheme for the design variables with meaningful genetic operators is important in genetic algorithm to obtain optimal design solutions with minimal computational effort.

In the previous paragraph, we showed that the performance of the genetic algorithm based composite design can be improved by proper use of solution representation. Another approach to enhance the performance focuses on discretization of the design space using decomposition of the actual problems into sub-problems [12, 13]. The composite structural optimization design problem has both real and discrete variables in the formulation. When these real and discrete variables are represented using a single string of GA then the cost of the search space increases. Therefore, decomposition approaches have are used to increase the efficiency [14, 15]. The optimization problem is decomposed into two levels. In the first level, the ply orientations are optimized for maximum material efficiency, while in the second level the laminate weight is minimized. The two-level optimization is solved using gradient based optimization in [14] and multi-level genetic algorithm in [15]. The multi-level genetic searches are performed sequentially with different populations and fitness functions for faster convergence and increased efficiency of the algorithm.

The selection/nature of the objective function is also an interesting problem in a composite structural design. Most of the research works in composite design problems address the minimization or maximization of certain structural characteristics such as weight, buckling load, stiffness and strength using single objective function [16–19]. However, in some studies, the objective is to design the composite structure for a specified design requirement rather than maximizing or minimizing its characteristics. Design of composite structures for a given stiffness/strength specifications is an interesting problem than maximizing or minimizing the structural characteristic. Recently, few research works have been carried-out to handle the multi-objective nature of structural design problem [20, 22].

Designing composite structures for given specifications is an approach that is used in aerospace industries. For example, design of box-beam cross sectional member of a helicopter rotor blade belongs to this category. In this case, the desired stiffness is specified using the aero-elastic optimization studies. The composite design problem addressed in this paper has two different objectives: first objective is to attain desired stiffness and the other objective is to maximize the elastic coupling. In this paper, we present a multi-agent search technique called 'Particle Swarm Optimization' (PSO) for solving the multi-objective composite structural design problem. Like GA's, PSO is also a population based stochastic optimization technique [23,24], inspired by social behavior of bird flocking or fish schooling. In PSO, each solution to the optimization problem is regarded as a 'particle' in the search space, which adjusts its position in the search space according to its own flying experience and the flying experience of other particle. Hence, PSO has high speed of convergence than the other evolutionary search algorithm [23, 24]. To study the performance of the proposed algorithm, we consider two cases of composite box-beam optimization. First, a composite box beam is designed to match the specified stiffness values with the elastic coupling terms being zero. Next, the box beam is optimized to match the specified stiffness values, and some of the elastic coupling terms are minimized simultaneously. The optimal geometry and ply angles are obtained for composite box-beams with 10°, 15° and 45° discretizations. The performance and computational efficiency of the PSO algorithm is compared with GA based approaches.

2 Problem Formulation

In aerospace vehicles the most commonly used beam structures are of type I-sections, Z-sections and box-beam cross sections. The aero-elastic optimization studies of these beam-type structures use the 1-D effective elastic stiffness values. Designing the actual cross section for the optimal 1-D effective elastic stiffness values obtained from an aero-elastic optimization study is a complicated problem.

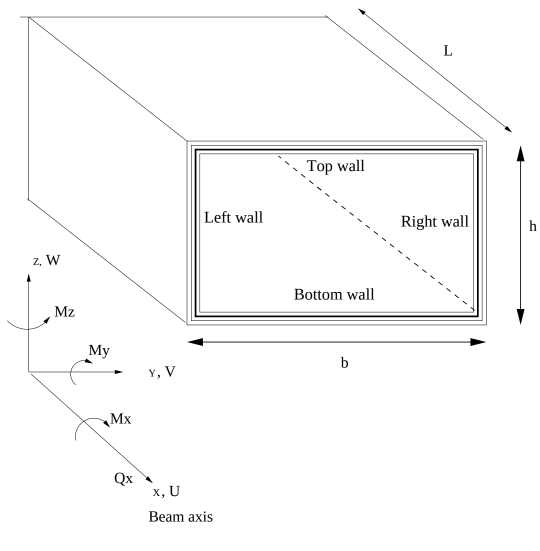

Figure 1: Composite box-beam configuration

In this paper, a composite box-beam design optimization is presented such that the optimal box-beam configuration satisfies the desired stiffness requirements with the maximum elastic coupling. The desired beam stiffness values (S) are obtained from the aero-elastic optimization study that uses one-dimensional beam stiffness values as design variables. An analytical composite box-beam formulation presented in [25] is used for predicting the effective elastic stiffness of the composite box-beam. A typical composite box-beam geometry and its coordinates are shown in Fig. 1. The deformation of the box-beam is described by three transverse displacements u, v and w, and a torsional displacement ϕ. The cross-sectional stiffness matrix of a composite box-beam is

$$\left\{ \begin{array}{c} Q_x \\ M_x \\ -M_y \\ M_z \end{array} \right\} = \left[\begin{array}{cccc} K_{11} & K_{12} & K_{13} & K_{14} \\ K_{12} & K_{22} & K_{23} & K_{24} \\ K_{13} & K_{23} & K_{33} & K_{34} \\ K_{14} & K_{24} & K_{34} & K_{44} \end{array} \right] \left\{ \begin{array}{c} u_e^{'} \\ \phi^{'} \\ w^{''} \\ v^{''} \end{array} \right\} \tag{1}$$

The off-diagonal terms in the above stiffness matrix K are referred as elastic couplings that play a role in the dynamic response of the composite box-beam structure.

Now, let S be the desired stiffness value for a box-beam. The stiffness requirement is derived from an aero-elastic optimization study [26] that is represented as:

$$S = \left[\begin{array}{cccc} K_{11}^T & K_{12}^T & K_{13}^T & K_{14}^T \\ K_{12}^T & K_{22}^T & K_{23}^T & K_{24}^T \\ K_{13}^T & K_{23}^T & K_{33}^T & K_{34}^T \\ K_{14}^T & K_{24}^T & K_{34}^T & K_{44}^T \end{array} \right] \tag{2}$$

where the superscript T represent the target values. The composite box-beam has to be designed such that the stiffness values of the box-beam matches with the desired stiffness values S.

One of the main advantages of using composite material is to tailor the structure for beneficial elastic couplings. These elastic couplings can be introduced by designing

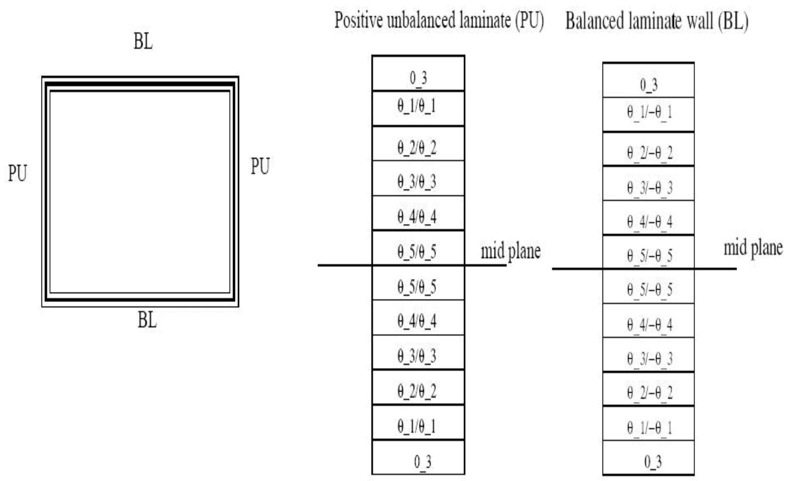

Figure 2: Balanced and Unbalanced configuration of composite box-beam

the composite box-beam with the unbalanced walls as shown in Fig. 2. In helicopter aero-elastic optimization studies, the rotor blade is often idealized as a composite box-beam. The term K_{24} in the stiffness matrix is responsible for the lag bending torsion coupling. The lag-bending torsion coupling is beneficial in increasing the aero-elastic stability of the main rotor. A positive value of K_{24} increases stability whereas a negative value decreases stability. This elastic coupling K_{24} can be introduced by unbalancing the opposite walls of the box-beam as shown in Fig. 2. The box-beam stiffness matrix for this case can be written as

$$\left\{ \begin{array}{c} Q_x \\ M_x \\ -M_y \\ M_z \end{array} \right\} = \left[\begin{array}{cccc} K_{11} & 0 & 0 & 0 \\ 0 & K_{22} & 0 & K_{24} \\ 0 & 0 & K_{33} & 0 \\ 0 & K_{24} & 0 & K_{44} \end{array} \right] \left\{ \begin{array}{c} u_e^{'} \\ \phi^{'} \\ w_b^{''} \\ v_b^{''} \end{array} \right\} \quad (3)$$

The axial stiffness, i.e., K_{11}, in the above matrix does not play a major role in the dynamics of the beam structure as its magnitude is very high compared to the other stiffness values. Thus, the objective of this paper is to design a composite beam to achieve the desired bending and torsional stiffness values (K_{ii}^T, $i = 2, 3, 4$) with maximum elastic coupling K_{24}.

The stiffness of the box-beam changes with change in geometry of the box-beam (b and h), and the stacking sequence Θ. The stacking sequence or the ply angle orientations represent the arrangement of plies in each wall of the box-beam. For unbalanced configuration, the stacking sequence for horizontal and vertical walls varies. Hence, the stacking sequence for unbalanced configuration is defined as

$$\Theta = \left[\left(\theta_1^{BL}, \theta_2^{BL}, \cdots, \theta_n^{BL}\right), \left(\theta_1^{PU}, \theta_2^{PU}, \cdots, \theta_n^{PU}\right)\right] \tag{4}$$

where n is the number of plies in each laminate. The geometry and stacking sequence of the box-beam are taken as design variables. The design variables can be represented as

$$\mathbf{x} = [b,\ h,\ \Theta] \tag{5}$$

The design variables are subjected to geometric constraints that are represented as

$$\begin{array}{ll} \textbf{Constraint 1} & b^l \leq b \leq b^u \\ \textbf{Constraint 2} & h^l \leq h \leq h^u \\ \textbf{Constraint 3} & \Theta \in \{o_1, o_2, \cdots, o_m\} \end{array}$$

where l represents lower bound while the upper bound is denoted by u, and o is the available discretization to laminate.

The multi-objective composite rotor blade design problem is gives as:

$$\textbf{Objective 1} \quad \textit{Minimize} \quad F_1(\mathbf{x}) = 100\left|\frac{K_{22} - K_{22}^T}{K_{22}^T}\right| \tag{6}$$

$$\textbf{Objective 2} \quad \textit{Minimize} \quad F_2(\mathbf{x}) = 100\left|\frac{K_{33} - K_{33}^T}{K_{33}^T}\right| \tag{7}$$

$$\textbf{Objective 3} \quad \textit{Minimize} \quad F_3(\mathbf{x}) = 100\left|\frac{K_{44} - K_{44}^T}{K_{44}^T}\right| \tag{8}$$

$$\textbf{Objective 4} \quad \textit{Maximize} \quad F_4(\mathbf{x}) = K_{24} \tag{9}$$

The objectives (1,2, and 3), describe the closeness of the stiffness parameters of the composite beam with respect to the desired values while objective 4 describes the stability of the helicopter rotor. Recently, some researchers have used genetic algorithms [27–30], to solve multi-objective optimization problems. In [30] a survey on different methods for multi-objective optimization using evolutionary algorithms is presented.

The multi-objective problems are solved by combining the objective functions into a single function. For example, a) Min-Max method b) weighted sum method and c) goal programming are most commonly used for combining objective functions. One has to select the function such that the minimization of a single function guarantee simultaneous minimization of other objective functions. In our problem, the desired values are specified for the first three objectives hence they are combined into a single function using Min-Max method. Since there is no desired value specified for K_{24}, it is difficult to embed the objective functions into a single objective function. Also, in composite design problem, the first objective is to achieve the desired stiffness and than maximize the coupling. Hence, a single objective function for the optimization problem is formulated as

$$Minimize, \quad J(\mathbf{x}) = \begin{cases} -K_{24}, & if\ J_1 \leq \epsilon \\ J_1, & otherwise \end{cases} \tag{10}$$

where $J_1 = Max\,(F_1, F_2, F_3)$. The parameter ϵ is the maximum allowable tolerance in the stiffness value.

Using the above defined objective function and geometric constraints the design problem is formulated as:

Minimize $J(\mathbf{x})$

Subject to **Constraint 1-3**

The optimization problem is combinatorial by nature and is *NP*-hard. To solve this combinatorial optimization problem a particle swarm optimization algorithm is used.

3 Particle Swarm Optimization

The particle swarm optimization (PSO) algorithm belongs to the category of swarm intelligence techniques. The swarm intelligence concepts are inspired by the social behavior of flocking animals such as swarms of birds, ants and fish school. PSO was first developed and introduced as a stochastic optimization algorithm by Eberhart and Kennedy [23]. PSO is a recently developed heuristic technique, inspired by the choreography of a bird flock. The approach can be viewed as a distributed behavioral algorithm that performs a multidimensional search. PSO has been found to be useful in a wide variety of optimization tasks. Due to its natural ability to converge faster, PSO algorithm is also used to solve multi-objective optimization problems [24].

PSO is a population based algorithm that exploits a population of individuals to probe promising regions of the search space. The individual behavior is affected either by the best local or best global individual. The performance of each individual is measured using fitness function similar to evolutionary algorithms. The population is referred as a swarm and individuals are called particles. The particles move in a multidimensional search space with adaptable velocity. In PSO, the particles remember the best position in the past and the best position ever attained by the particles. This property helps the particles to search the multidimensional space faster.

Let us consider an optimization problem with n-dimensional design space. Assume that there are M particles in a swarm and i^{th} particle in a swarm is represented as a vector X_i, $X_i \in \Re^n$:

$$X_i = (x_{i1}, x_{i2}, \cdots, x_{in})^T, \quad i = 1, 2, \cdots, M \tag{11}$$

The velocity of the particle moving in the n-dimensional search space is

$$V_i = (v_{i1}, v_{i2}, \cdots, v_{in})^T, \quad i = 1, 2, \cdots, M \tag{12}$$

and the best position encountered by the particle is

$$B_i = (b_{i1}, b_{i2}, \cdots, b_{in})^T, \quad i = 1, 2, \cdots, M \tag{13}$$

Let us assume that the particle j attains the best position in the current iteration (l) then the position and the velocity of the particles are adapted using the following equations.

$$V_i(l+1) = wV_i(l) + c_1 r_1 (B_i(l) - X_i(l)) + c_2 r_2 (B_j(l) - X_i(l)) \tag{14}$$

$$X_i(l+1) = X_i(l) + V_i(l+1) \tag{15}$$

where w is the inertia weight, c_1, c_2 represent positive acceleration constants and r_1, r_2 are uniformly distributed random numbers $r_1, r_2 \in [0, 1]$. The first term in the above equation, relates to the current velocity of the swarm, the second term represents the local search while the third term represents the global search pointing towards the optimal solution.

The inertia weight (w) is employed to control the impact of the previous history of velocities on the current velocity of each particle. Thus, the parameter w regulates the trade-off between global and local exploration ability of the swarm. A general rule of thumb suggests that it is better to initially set the inertia to a large value, in order to make better global exploration of the search space and gradually decrease the weight to get more refined solutions. Thus, a time decreasing inertia weight value is used in this paper.

The algorithm for particle swarm optimization can be summarized as follows:

- Let l=1; Initialize the position and velocity of the particles in a swarm.
- Evaluate the performance of each particle.
- Store the best position of each particle and best position in a swarm.
- WHILE the maximum number of iteration has not been reached DO
  - Update the velocity and position using the equations (14) and (15).
  - Maintain the particles within the search space in case they go beyond its boundaries. This condition ensures a valid solution for a given problem.
  - Evaluate the performance of each particle in a swarm
  - Store the best position of particles and swarm
- Increment the iteration loop by one, i.e., $l = l + 1$.
- END WHILE

The selection of acceleration constants and random variables affect the convergence of the PSO algorithm. A detailed study on theoretical investigations of parameter selection and convergence properties of PSO algorithm is provided in [31, 32].

4 Experimental Results

A Simulation study was carried out on two different cases of composite box beam designs, namely balanced wall condition (no elastic coupling) and unbalanced walls. The box-beam design problem is solved using PSO and genetic algorithm approaches.

In genetic algorithm approach, the design space is represented using the real numbers and is referred as real-coded genetic algorithm (RCGA). The design problem is also solved using binary-coded genetic algorithm (BGA) approach. The parameters used for genetic algorithm approaches are shown in Table. 1.

Table 1: Genetic algorithm parameters

Parameters	RCGA
Population	30
Generations	500
Crossover probability	0.6
Mutation probability	0.05

4.1 Box-Beam Configuration

A single celled composite box-beam is optimized to provide the specified stiffness values. These stiffness values are obtained from an aero-elastic optimization study in which the 1-D beam stiffness values were used as the design variables [26]. An initial box-beam configuration which provides reasonable cross sectional properties of the 1-D beam used in the aero-elastic optimization study [25] is selected. The selected box-beam configuration is optimized to provide the specified structural stiffness. The initial design has an outer box width of 4.2in. and height of 2.2in. Each wall of this box-beam have 26 plies, each ply having a thickness of 0.005in. The plies are made of graphite epoxy ($AS413501-6$) and its material properties are given in Table 2. The target stiffness values are given in Table 3.

In the optimization process, each wall of the box-beam is considered to have 26 plies. Considering the mid-plane symmetry of the laminate, the maximum number of ply angle design variables reduces from 26 to 13. Three plies at the outer edge of the wall are fixed with 0^o plies to provide the required axial stiffness. Therefore, out of

Table 2: Graphite/Epoxy : Material Properties

$E1$(GPa)	141.5
$E2$(GPa)	9.8
$G12$ (GPa)	5.9
ν	0.42
ρ (Kg/m^3)	1445.4

13 plies, only five ply angles are considered as design variables for optimizing the box-beam. The stacking sequence of each wall of the box-beam can be written as

$$[0_3, \pm\theta_1, \pm\theta_2, \pm\theta_3, \pm\theta_4, \pm\theta_5]_s \tag{16}$$

Table 3: Target Stiffness Values

Parameter	Stiffness in $N - m^2$
K_{22} (GJ)	20419.79
K_{33} (EI_y)	38364.46
K_{44} (EI_z)	82916.73

Assume there are N choices for each ply angle design variable, hence there can be N^5 possible laminate designs. The allowable discrete values for ply angle design variables are given in Table 4. Three cases of discretization are considered for the experimental study. In discretization I, the allowable ply angles are integer multiples of 10° between 0° and 90°. Similarly, in discretization II and III, the allowable ply angles are integer multiple of 15° and 45°. The geometry variables, breadth b and height h, are constrained to have an upper and a lower bound to avoid an infeasible box-beam configuration. The geometric constrains are: $3 \leq b \leq 5$in and $2 \leq h \leq 3$in.

Now, we present the results obtained using the proposed methodology for composite box-beam design with and without elastic coupling. First, we present the results for without elastic coupling.

Table 4: Ply Angle Discretization

Discretization I	[0,10,20,30,40,50,60,70,80,90]
Discretization II	[0,15,30,45,60,75,90]
Discretization III	[0,45,90]

4.2 Case I: Without elastic coupling

For this case, consider a balanced box-beam configuration, i.e., the four walls have similar configuration. Hence, the elastic coupling term K_{24} is zero. As mentioned earlier, from the given discretization only 5 ply angles need to be selected. The ply angle vector Θ for optimization is given as

$$\Theta = [\theta_1, \theta_2, \theta_3, \theta_4, \theta_5] \tag{17}$$

The two real design variables (b and h) and five integer variables (Θ) are needed to be optimized. Hence, the particle i in the swarm is represented by a 7-dimensional vector X_i as:

$$X_i \quad = \quad (b_{i1}, h_{i2}, \theta_{i3}, \theta_{i4}, \cdots, \theta_{i7}) \tag{18}$$

Let $[o_1, o_2, \cdots, o_m]$ be m possible ply angles specified by the user (see constraint 3) while the integer variables (θ_{ij}, $j = 3, 4, \cdots, 7$) are coded as a real variable but they are converted into integers using the equation given below:

$$\hat{\theta_{ij}} = o(k) \quad where \quad k = arg \min_{k=1,2,\cdots,m} \|o_k - \theta_{ij}\|, \quad j = 1, 2, \cdots, 5 \tag{19}$$

The acceleration constants are selected as $c_1 = 1$, $c_2 = 2$ and random constant is gradually decreased from 0.8 to 0.1 with a linear decreasing rate. The maximum velocity V_{max} is taken as the dynamic range of the particle in each iteration. The PSO and RCGA based composite box-beam design methodologies are implemented in MATLAB on a Pentium IV machine. The simulation was carried out using PSO and GA for five times with each discretization and the best results are reported in Table. 5. The objective function J value, and average error between the target stiffness values and actual box-beam stiffness values are also reported for the three different discretization cases.

From the table, one can see that the PSO based approach provides better design solution than the RCGA approach. The average error for PSO design in discretization I is approximately two times less than the average error for RCGA design. The same can also be observed in other discretization cases. It is also observed from the table that the box-beam dimensions obtained using PSO for three discretization cases are almost same. In case of RCGA approach, the box-beam dimension increases substantially for discretization II and III. Even with increase in the dimension, the RCGA approach was not able to find the best solution. This clearly shows the ability of PSO algorithm to find best possible solution for a given design problem.

A common practice in stacking sequence optimization is to avoid the consecutive appearance of more than four plies in a laminate to overcome matrix cracking. This restriction is generally used as a constraint in the laminate optimization problems. From Table. 5, the optimal solution given by PSO always satisfies the ply angle constraint, where as the RCGA approach does not satisfy the constraint for discretization III. In discretization III, the design variables θ_4 and θ_5 have 90^o ply angles which leads to the appearance of eight consecutive plies having the same ply angle according to the laminate configuration given in Equation 16. Therefore, the design solution obtained for discretization case III by RCGA is discarded as a invalid design.

From the average error given in the table, the PSO algorithm is able to find a

Table 5: Design solutions for three discretization : No elastic coupling

	PSO			RCGA		
	Discretization			Discretization		
Variables	I	II	III	I	II	III
Breadth (inch)	3.78	3.82	3.82	3.69	4.35	4.36
Depth (inch)	2.32	2.35	2.23	2.27	2.58	2.65
θ_1	30	45	45	40	90	90
θ_2	50	90	0	50	90	90
θ_3	80	15	45	0	45	45
θ_4	20	75	90	40	75	90
θ_5	70	45	45	90	90	90
Objective function, J	0.15	0.78	4.50	0.25	2.55	5.08
Average error (%)	0.09	0.51	4.13	0.19	1.85	4.60

solution very close to the optimal configuration for the discretizations I and II. Based on the box-beam dimension and objective function, the optimal solution obtained for discretization I is the best design solution for the helicopter rotor blade design problem.

4.3 Case II: With elastic coupling

The box-beam configuration shown in Figure. 2 produces a lag-bending torsion coupling. In this case, the box-beam is designed for maximum elastic coupling term K_{24} while satisfying the desired stiffness values given in Table. 3. In the design problem, once the error between the desired stiffness values and the actual box-beam stiffness values are less than the maximum allowable tolerance ϵ, then the optimizer maximizes K_{24}. If the error between the actual box-beam and the specified stiffness value is greater than the ϵ, then the optimizers' objective is to achieve the desired stiffness values. The value of maximum allowable tolerance ϵ is selected as 2%.

The ply orientations of vertical wall are chosen to be 0^o plies on the outside to ensure the minimum bending stiffness and axial stiffness. The bending stiffness depends on the location of plies from mid plane of the composite laminate. The 45^o plies are kept towards the mid plane to ensure the shear stiffness [33]. The stiffness K_{24} introduces negative lag bending torsion coupling that increases the aero-elastic stability of the rotor blade. The stacking sequence of balanced laminate (BL) is written as

$$[0_3, \pm\theta_1, \pm\theta_2, \pm\theta_3, \pm 45_2]_s \tag{20}$$

Similarly, the stacking sequence of the unbalanced laminate (PU) is written as

$$[0_3, \theta_1, \theta_1, \theta_2, \theta_2, \theta_3, \theta_3, \pm 45_2]_s \tag{21}$$

Hence, the ply angle variables $\theta_1, \theta_2, \theta_3$ of the balanced laminate, ply angle variables $\theta_1, \theta_2, \theta_3$ of the unbalanced laminate and geometry configurations (b, h) are chosen as design variables. The ply angle vector Θ is defined as

$$\Theta \quad = \quad [(\theta_1, \theta_2, \theta_3)^{BL}, (\theta_1, \theta_2, \theta_3)^{PU}] \tag{22}$$

The simulations were carried out for five times using PSO and RCGA approaches. The best results are reported in Table. 6. For discretization I, both PSO and RCGA approaches were able to find the optimal solution, as shown in the table. In case of discretization II and III, the elastic coupling obtained using RCGA and PSO are almost similar. From the experimental results for PSO approach, the 20° and 15° ply angle orientations play a major role in maximizing K_{24} for discretization I and II. The magnitude of elastic coupling K_{24} and mass of the box-beam are almost equal for discretization I and II. Also, the elastic coupling obtained for discretization III is less than the other discretization. This is because, the design space for ply angle variables is less. A similar behavior can also be seen from the simulation results obtained using RCGA approach. From Table 6, we can see that the design solution obtained using PSO and RCGA are valid designs, i.e., four plies with same ply angle

Table 6: Design solutions for three discretization : Elastic coupling

	PSO			RCGA		
	Discretization			Discretization		
Variables	I	II	III	I	II	III
Breadth (inch)	3.43	3.58	3.62	3.43	3.52	3.74
Depth (inch)	2.27	2.32	2.38	2.27	2.17	2.31
θ_1^{BL}	20	15	90	20	15	90
θ_2^{BL}	20	45	90	20	30	90
θ_3^{BL}	70	15	45	70	15	45
θ_1^{PU}	10	30	45	10	30	90
θ_2^{PU}	20	15	90	20	15	45
θ_3^{PU}	20	15	45	20	15	90
K_{24}	7144	6675	2120	7144	6558	2070

never occurs in the best solutions. The most commonly preferred discretization in the aerospace industry is 45/90°.

4.4 Discussion

Some of the important characteristics of heuristic search techniques need to be discussed for solving the multi-objective design problems. For this purpose, a mean fitness of population/swarm, standard deviation from best solution and computational time required to find the best solution are considered on the box-beam design problem without elastic coupling maximization for discretization case I. The performance parameters of the proposed PSO approach is compared with the RCGA and binary coded genetic algorithm (BGA) techniques. The PSO and GA based techniques are implemented in MATLAB on a Pentium IV machine. A population of 20 individuals to search the design space is considered and the search process is terminated when the objective function (J) is less than or equal to 0.25% or maximum number of generation is equal to 2000.

The mean fitness of the population, standard deviation, number of times the best

Table 7: Simulation study on performance Evaluation

Method	Avg. fitness	Std. deviation	Avg. Term. gen.	R	T (min)
PSO	0.152	0.0563	356	12	32.34
RCGA	0.296	0.0668	624	14	42.35
BGA	2.014	1.9403	2000	2	101.27

fitness is achieved (R), termination generations and CPU time (T) are recorded and given in Table. 7. From the simulation results, we can see that the proposed PSO algorithm require lesser computational effort and also find best design solutions than the GA based approaches. It is also observed from simulations that BGA requires more generations to find the best solution. This fact is reflected in the mean and standard deviation of the population. Hence, from the convergence study, the proposed PSO algorithm is more suitable for structural design problems.

5 Conclusion

A multi-agent search technique called particle swarm optimization is presented for multi-objective composite box-beam design to a helicopter rotor blade. The main objective of the composite box-beam design problem is to find the optimal geometry and stacking sequence of the structure such that it satisfies the desired stiffness requirements and also has maximum elastic coupling. The multi-objective and combinatorial nature of the design problem is solved using particle swarm optimization approach. In this paper, a *min-max* strategy is used to convert the three desired stiffness objectives into a single objective. The *min-max* strategy and maximization of elastic coupling are collectively considered to design a composite box-beam. The design solutions are presented for different discretization (10°, 15° and 45°) and are compared with other genetic algorithm based approaches. The performance in terms of closeness to target values and the computational time obtained using PSO approach is better than the genetic algorithm based approaches. The experimental results clearly shows that the PSO approach always provides a better and valid design solution than the genetic algorithm approaches.

Acknowledgement:- The authors would like to thank Prof. Ganguli, Department of Aerospace Engineering, Indian Institute of Science, India for his support.

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