# REGULARIZED SPECKLE REDUCING ANISOTROPIC DIFFUSION FOR FEATURE CHARACTERIZATION

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## ABSTRACT

For tissue characterization in medical ultrasound imagery or terrain characterization in synthetic aperture radar imagery, it is necessary to preprocess imagery to reduce granular, texture-alike noise called speckle. This preprocessing is difficult when it is needed to preserve delicate image details that are buried in speckle. Speckle reducing anisotropic diffusion (SAR) is a partial differential equation-based method developed for this purpose. Toward its improved performance for point/linear features, we introduced a novel regulator called energy condensation integral and developed a regularized SRAD (Reg-SRAD) via minimization. The Reg-SRAD generates outputs with increased resolution for point and linear features while retaining the characteristics the SRAD-filtering speckle with regional features enhanced. The performance of the method has been illustrated using synthetic and real ultrasound data, and radar imagery as well.

*Index Terms* – speckle, filtering, synthetic aperture radar, diffusion equations, regulators

### **1. INTRODUCTION**

Speckle plagues the visualization and interpretation of fine structures in coherently imaged imagery such as ultrasound (US) and synthetic aperture (SAR) radar images. The challenge in speckle filtration is how to achieve desired speckle removal with least loss of details. Many speckle filters [6, 7, 8] have been developed with the capability of retaining certain features. However, these techniques do not meet the need of quantitative image analysis for tissue or terrain characterization. More delicate feature-preserving speckle filters are in need and being sought.

Speckle reducing anisotropic diffusion (SRAD) [12] has been emerged as a tool for reducing speckle with regional feature neatly enhanced. The method relies on the *instantaneous coefficient of variation* (ICOV) edge detector [12, 13] as a controller of diffusion rate near edges of regional structures. In SAR imagery, man-made targets usually show as dominant point scatterers and it is necessary for object detection and recognition methods attempt to extract the locations of them. In ultrasound images of artery, linear features are important. For thin linear features and point features, SRAD tends to broaden (see Fig. 1). Though for regional feature characterization, the broadening is trivial; for linear feature characterization, it needs to be corrected. There exists a need to improve SRAD for applications where point and linear feature are critical

To alleviate point/linear feature broadening problem, a DeSpeRADo [1] method is proposed that combines the strength of SRAD and deconvolution restoration. This technique assumes that feature distortion is caused by the convolution of the point spread function (PSF) of the imaging system with the underlying feature. Hence, Deconvolution sharpens features, while SRAD is removing speckle. DeSpeRADo showed promising results on synthesized ultrasound data, although no results were reported for real data. However, the DeSpeRADo assumes a linear signal model for envelop-detected amplitude signals - an assumption that topples general belief that such linear signal model only exists for radio frequency (RF) data [11]. Moreover, the estimation of system and environmentinduced PSF has remained to be an unsolved, challenging problem [11]. These limitations may hinder the use of DeSpeRADo in real applications of feature characterization.

It is desirable to pursue methods that do not require the use of PSF for correcting feature broadening distortion. The method to be presented in this paper is inspired by recent advances in high resolution SAR image formation techniques based on nonquadratic ( $l_k$ -norm) regularization [2, 4]. In particular,  $l_k$ -norm imposes an energy-type constraint on the reconstructed solution, and shows the capability to suppress artifacts and increase the resolvability of scatterers. This class of methods has been shown promising to generate super-resolution images with point/linear features enhanced, relative to the SAR images formed by conventional Range-Doppler algorithms [3, 5]. The  $l_k$ -norm regularization technology, while has the capability to increase resolution, does not rely on the PSF.

In this paper, we incorporate the nonquadratic regularization into SRAD in order to enhance the performance of SRAD for point/linear feature characterization; and derive a Reg-SRAD partial differential

equation from the perspective of energy functional minimization. The resultant diffusion equation enables the correction of feature broadening distortions with minimum operations added.

## 2. METHOD AND THEORY

The Reg-SRAD may serve as a general PDE-based approach for removing speckle and enhancing point/linear/regional features. It is applicable to either ultrasound, SAR, or Lidar intensity imagery. The Reg-SRAD partial differential equation (PDE) is composed of two components: the SRAD diffusion component and the energy contraction component. The former accounts for speckle removal and regional feature enhancement; and the latter reduces the broadening distortion of point and/or linear features.

For notational simplicity, let vector  $\vec{x}$  denote any location in image domain O. The SRAD partial differential equation can be written as follows:

$$I_t(\vec{x};t) = \nabla \cdot [c(q)\nabla I(\vec{x};t)], \qquad (1)$$

where  $I_t \equiv \partial I / \partial t$ ,  $\nabla$  is the gradient operator,  $\nabla \cdot$  is the divergence operator, q is the *instantaneous coefficient of* 

variation defined by

$$q(\vec{x},t) = \frac{\left| (1/2) \left\| \nabla I(\vec{x},t) \right\|^2 - (1/16) \left[ \nabla^2 I(\vec{x},t) \right]^2 \right|^{1/2}}{\left[ I + (1/4) \nabla^2 I(\vec{x},t) \right]}, \quad (2)$$

where  $\nabla^2$  is the Laplacian operator,  $\|\cdot\|$  the magnitude of gradient, and  $|\cdot|$  the absolute value. The scalar function c(q) in (1) is calculated by

$$c(q) = \frac{1}{1 + [q^2(\vec{x}, t) - q_0^2(t)] / [q_0^2(t)(1 + q_0^2(t))]} .$$
 (3)

where  $q_0(t)$  is the coefficient of variation measured in a homogeneous speckle area at instant t. The PDE (1) is subjected to the initial condition  $I(\vec{x}; t = 0) = I_0(\vec{x})$  (with  $I_0(\vec{x})$  being the input image) and mirror boundary

 $T_0(x)$  being the input image) and mirror boundary condition.

We view (1) to be the evolution equation of an Euler equation that is derived from an energy functional minimization problem given by:

$$E_0(I) = \int_{\Omega} f(q) d\vec{x} \tag{4}$$

where function f(.) is nonnegative and increase and q is the *instantaneous coefficient of variation* as defined in (2). In fact, in [14] it can be seen that (1) can indeed be derived approximately from (4) as far as the diffusive function (3) is not specified as such. Now, (4) is the starting point for our derivation of the Reg-SRAD. In order to emphasize point and linear features that are not explicitly treated by SRAD, we introduce an energy-condensation regulator in the SRAD energy functional (4), forming the dual-objective energy functional as follows:

$$E(I) = E_0(I) + \lambda \int_{\Omega} (I/I_c)^{\gamma} d\vec{x}$$
(5)

where  $\lambda$  is a positive weight factor,  $\gamma$  a parameter greater than unity, and  $I_c$  a threshold value above which image features are considered bright. The first term in (5) gets diminished through diffusion. The regulator (second term) imposes an energy-condensation constraint for bright features on the diffusion solution. With a  $\gamma >> 1$  one observes that: 1) the broadening of bright image features (compared to  $I_c$ ) during the diffusion process would increase the total energy in (5) rapidly, and 2) the majority of image regions (that are darker than  $I_c$ ) undergoes SRAD essentially as  $(I/I_{a})^{\gamma}$  approaches to zero when  $I < I_{c}$ . Therefore, the regulator in (5) serves to prevent the fattening of bright spots/linear structures without noticeably affecting normal SRAD diffusion in dominant image regions. A larger  $\gamma$ value favors a solution with increased resolvability of bright point and linear features. Weight  $\lambda$  determines the emphasis on the speckle smoothing and point/linear feature preservation.

By minimizing the energy functional (5), we have derived the following Reg-SRAD evolution equation:

$$I_t(\vec{x};t) = \nabla \cdot \left[ c(q) \nabla I(\vec{x};t) \right] - \lambda \gamma I_c^{-\gamma} I^{\gamma-1}(\vec{x},t) .$$
(6)

To avoid trivial solution of all zeros, we restrict ourselves to solutions of (6) in the space of equal energy functions (i.e.,  $\int_{\Omega} I^{\gamma} d\vec{x} = \int_{\Omega} I_{\Omega}^{\gamma} d\vec{x}$ ). To solve (6), three constants,  $\lambda$ ,  $\gamma$  and  $I_c$ , must be determined. In principle, weigh  $\lambda$  can be found by the technique in [9]. However, there is no rule to choose the gamma parameter. Empirically, we choose a gamma value in the range of  $1.2 \sim 3$ . A value of  $I_c$  is adaptively set as the median (or mean) of the image function I. The partial differential equation in (6) can be easily digitized using a differencing scheme and solved by iterative method. After each iteration, the energy of the updated I is rescaled by a factor of  $\langle I_0 \rangle / \langle I \rangle$  (where  $\langle \rangle$  denotes mean value) so that the processed image has the same energy as the input  $I_0$ . Starting from the original speckle image, it takes ~300 iterations for (6) to converge to a stationary solution for a time step  $\Delta t = 0.05$ . Finally, it is worthwhile to point out that the Reg-SRAD can be solved equivalently by reversing the sign of  $\lambda$  and letting  $\gamma < 1$ . With the equivalent method, a gamma value of 0.75 gives satisfactory results in most cases. The equivalent method is appropriate for hardware implementation of the Reg-SRAD algorithm.

#### **3. RESULTS AND DISCUSSION**

To show the performance of the Reg-SRAD method, the results from experiments using a synthesized ultrasound image were first evaluated in terms of feature preservation and speckle/artifacts reduction.

The synthesized, test ultrasound image contains four objects-a circular, two intersecting line features and one point. The mean reflectivity image is shown in Fig. 1(Top left). We model randomly-distributed micro-scatter within the reflectivity image as a Gaussian-distributed field of zeromean and unity variance. By pointwisely multiplying the mean reflectivity and the Gaussian field, we get the reflectivity image. This reflectivity function is then convolved with a two-dimensional ultrasound pulse or point spread function, yielding the radio frequency data. The test ultrasound image (amplitude) is taken as the envelopdetected RF data obtained by adding a Hilbert transform of the RF signal and taking the absolute value of the complex signal. Figure 1 (top right) shows the synthesized ultrasound image after logarithmic transformation in order for better visualization. Note speckle everywhere and artifacts in vertical direction.

Example test results of the SRAD and the Reg-SRAD are shown in Fig. 1(lower left and right), respectively. The same number of iterations is used in both algorithms. In the SRAD result, speckle and artifacts have been smoothed adequately within all features. There are some cloudy artifacts in the background. Though all features have been enhanced in terms of contrast, the sizes of features become expanded. The broadening of point and linear features is more pronounced than that of the regional feature. On the contrary, in the Reg-SRAD result, artifacts and speckle in the background have been smoothed completely. However, within features there is remaining speckle. The broadening distortion problem of features has been correctly solved. The improved result allows for thin feature characterization.

Next, we demonstrate the improved performance of the Reg-SRAD method over the SRAD using real ultrasound images of human carotid artery. Fig. 2 (top left) shows tested image with speckle. The SRAD processed result is shown in Fig. 2 (top right), with speckle reduced and feature revealed. Though the feature broadening is not obvious there are some staircase artifacts. On the other hand, the Reg-SRAD result shows improvement in terms of speckle reduction and artifacts reduction.

Finally, we give example results of the Reg-SRAD algorithm for processing SAR images extracted from "Moving and stationary target acquisition and recognition public data set"-MSTAR data set [10] in Fig. 3. As expected, results are visually appealing, as the processed images appear as optical imagery.

In conclusion, a point-wise energy-condensation regulation technique has been developed to enhance the capability of SRAD algorithm for better preserving bright point and linear features. The new component in the Reg-SRAD partial differential equation is derived from the minimization of the regulator. The resulting result from Reg-SRAD strikes a balance between speckle reduction and point/linear/regional features emphasis. The Reg-SRAD improves the resolution of the image and enables the image to be more appealing to human visual systems.

#### 4. REFERENCES

- S. T. Acton, "Deconvolutional speckle reducing anisotropic diffusion," *IEEE International Conference on Image Processing*, Volume 1, pp. 5 – 8, 11-14 September 2005.
- [2] B. Borden, "Regularization of noisy ISAR images containing extended features," *IEEE trans. On Image Processing*, vol. 8, pp. 124-127, Jan. 1999.
- [3] Ian G. Cumming and John R. Bennett, "Digital processing of SEASAT SAR data," In *Record of the IEEE 1979 International Conference on Acoustics, Speech and Signal Processing*, Washington, B.C., 2-4 April 1979.
- [4] M. Cetin and W. C. Karl, "Feature-enhanced synthetic aperture radar image formation based on nonquadratic regularization," *IEEE Trans. Image Processing*, vol. 10, no. 4, pp. 623-631, 2001.
- [5] J. C. Curlander and R. N. McDonough, Synthetic Aperture Radar: Systems and Signal Processing, John Wiley and Sons, 1991.
- [6] V. S. Frost, J. A. Stiles, K. S. Shanmugan, and J. C. Holtzman, "A model for radar images and its application to adaptive digital filtering of multiplicative noise," *IEEE Trans. Pattern Anal. Machine Intell.*, Vol. PAMI-4, pp. 157-165, 1982.
- [7] D. T. Kuan, and A. A. Sawchuk, T. C. Strand, and P. Chavel, "Adaptive restoration of images with speckle," *IEEE Trans. on Acoustic. Speech Signal Processing*, Vol. ASSP –35, pp. 373 – 383, 1987.
- [8] J. S. Lee, "Digital image enhancement and noise filtering by using local statistics," *IEEE Trans. on Pattern Analysis and Machine Intelligence*, Vol. PAM1-2, 1980.
- [9] M. A. Gennert and A. L. Yuille, "Determining the optimal weights in multiple objective function optimization," *International Conference on Computer Vision*, December 5-8, 1988, pp. 87-89.
- [10] Air Force Research Laboratory, Model Based Vision Laboratory, Sensor Data management System MSTAR Web Page:http://www.mbvlab.wpafb.af.mil/public/sdms/datasets/m aster/.
- [11] Torfinn Taxt and Jarle Strand, "Two-Dimensional Noise-Robust Blind Deconvolution of Ultrasound images," *IEEE Trans. on Ultrason Ferroelectr Freq Control.*, 48 (4), pp. 861-866, July 2001.
- [12] Y. Yu and S.T. Acton, "Speckle reducing anisotropic diffusion," *IEEE Trans. on Image Processing*, vol. 11, pp. 1260-1270, November 2002.
- [13] Y. Yu and S.T. Acton, "Edge detection in ultrasound imagery using instantaneous coefficient of variation," *IEEE Trans. on Image Processing*, vol. 13, no. 12, pp. 1640-1655, December 2004.
- [14] Y. Yu, J. A. Molloy, S. T. Acton, "Generalized Speckle Reducing Anisotropic Diffusion for Ultrasound Imagery," *Proceedings of the 17th IEEE Symposium on Computer-Based Medical System*, Bethesda, Maryland, pp. 279-284, 24-25 June 2004.



**Fig. 1**. Results with synthesized data. Mean reflectivity function (Top left), Speckle US image (top right), SRAD result (Lower left), and Reg-SRAD result (lower right).



(3a) Original



(3b) Processed by Reg-SRAD



**Fig. 2**. Results from US data of carotid artery. Original US image (top); SRAD result (Lower left); and Reg-SRAD result (lower right).





(3d) Processed by Reg-SRAD Fig. 3. Results from two SAR images.