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Nonparametric Hammerstein Model Based Model Predictive Control for Heart Rate Regulation

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Abstract— **This paper proposed a novel nonparametric model based model predictive control approach for the regulation of heart rate during treadmill exercise. As the model structure of human cardiovascular system is often hard to determine, nonparametric modelling is a more realistic manner to describe complex behaviours of cardiovascular system. This paper presents a new nonparametric Hammerstein model identification approach for heart rate response modelling. Based on the pseudo-random binary sequence experiment data, we decouple the identification of linear dynamic part and input nonlinearity of the Hammerstein system. Correlation analysis is applied to acquire step response of linear dynamic component. Support Vector Regression is adopted to obtain a nonparametric description of the inverse of input static nonlinearity that is utilized to form an approximate linear model of the Hammerstein system. Based on the established model, a model predictive controller under predefined speed and acceleration constraints is designed to achieve safer treadmill exercise. Simulation results show that the proposed control algorithm can achieve optimal heart rate tracking performance under predefined constraints.**

Index Terms— **Hammerstein model identification, Model Predictive Control, Nonparametric model, Support Vector Regression, Heart rate control.**

I. INTRODUCTION

There are well-developed theories for the control and identification of linear time invariant (LTI) systems. However, as physical systems are nonlinear in nature, control and identification approaches for nonlinear system are practically important. A modest extension of linear model is Hammerstein model. The Hammerstein model can be described as a static nonlinear block followed by a dynamic linear system. Hammerstein models may account for nonlinear effects encountered in not only industrial processes but also physiological processes [1] [2].

The modelling of Hammerstein model is an active research topic [3] [4] [5] [6]. As far as the amount of priori information about the system is concerned, identification problems are either parametric or nonparametric [5]. In the nonparametric problem, priori information is much smaller than that of parametric problem. However, the nonparametric problem often better corresponds to real situations. For example, to model the response of cardiovascular systems, such as heart rate and respiration rate responses for exercise, the nonparametric identification approach should be more realistic because of the smaller information compared with the complexity of human cardiovascular system. This paper presents a new nonparametric identification approach based on support vector machine [7] and stochastic method [3] [8] [9]. Specifically, pseudo-random binary sequences (PRBS) type of experiments is performed to decoupling the identification of linear dynamic part from nonlinearity (as suggested in [3]). The correlation analysis is used to obtain the impulse and step response of the linear dynamic part. The powerful ϵ-insensitivity SVM approach is adopted to model the nonlinearity.

In order to guarantee safer exercise for rehabilitation, model predictive controller is designed based on the identified Hammerstein models. One of the unique features of engineering control practice is the presence of operational constraints that limit the expected performance of the controlled system. These constraints arise from economic (quality) or safety considerations as well as from physical restrictions. The main advantages of model predictive control (MPC) is that it allows us to use the detailed knowledge of a process, in the form of a dynamic model, as an aid to controlling that process within the required constraints [10]. In this study, we are trying to regulate heart rate of treadmill exercisers for rehabilitation. The constrains on speed, acceleration, and gradient of treadmill should be considered due to safety consideration and equipment limitations.

MPC has been well developed for linear systems. However, the complexity of the predictive control problem increases significantly for nonlinear systems.

In the case of Hammerstein systems, the most commonly used control method is based on direct inversion of the static nonlinearity combined with existing linear control approaches [11] [12] [13]. This strategy is also adopted in this paper. Firstly, the approximation of the inversion of static nonlinearity is directly obtained by using ϵ-insensitivity SVM. Then, the model predictive controller is designed for the simplified linear model to achieve desired tracking performance under predefined constraints.

In sport training, medical diagnosis, rehabilitation and analysis of cardio respiratory kinetics, automated exercise testing systems have revealed their growing importance [14] [15] [16]. Another topic of this paper is the design of an automated heart rate regulation system for exercise on a

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motorized treadmill. The proposed identification and control approach was applied for the automated heart rate regulation system design.

The paper is organized as follows. The proposed identification and model predictive control approach are given in Section II. Section III describes the identification of Hammerstein model of heart rate response for treadmill exercises based on PRBS type experiments. Simulation studies for the regulation of heart rate of treadmill exercisers are also presented in Section III. Section IV concludes the paper.

II. Proposed modelling and control approach

In this paper, we use a Hammerstein model to dynamically describe the relationship between walking speed and heart rate variation. As mentioned in the introduction, the linear dynamic identification of Hammerstein models can be decoupled from that of nonlinear parts by using pseudo-random binary sequences [3] type experiments. However, the PRBS inputs often cannot excite the nonlinearity sufficiently. To identify the nonlinear part or its inverse, steady state experiments should be performed.

A. Modelling the inverse of the nonlinear function by using SVR

To transfer a Hammerstein system to a linear system, a pre-compensator can be applied as in [11] and [17].

For the identification of the inverse of nonlinearity, the so called ϵ-insensitivity SVR regression will be employed, which is convex and very efficient in terms of speed and complexity. The description of SVR regression is omitted due to space limitation. However, brief introduction of SVR regression can be found in our previous paper [17]. Details about SVR, such as the selection of radius ϵ of the tube, kernel function, and the regularization constant C, can be found in [18] [7].

It should be emphasized that, as we need to model the **inverse** of the nonlinear function f, the measured steady state output y (heart rate) will be used as the $input\, data$, and the input $\tilde{u}$ (treadmill speed) as the $output\, data$.

B. Identification of linear dynamic part

In [3], Bai showed that the identification of linear part of a Hammerstein model can be decoupled from nonlinear part with the help of the PRBS input. The reason is that any static nonlinearity can be exactly characterized by a linear function, when driven by PRBS inputs which have a binary nature.

When a PRBS input is employed for the identification of the Hammerstein system, as shown in equation (2.3) of [3], the identification of a Hammerstein model can be simplified as a linear identification problem. Furthermore, the correlation analysis method can be applied to identify impulse and step responses of the linear dynamic part.

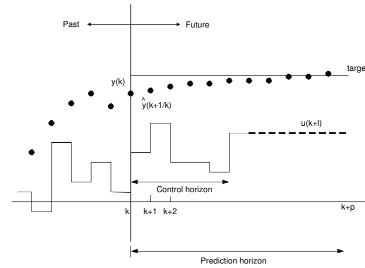

Fig. 1. Model predictive algorithm description.

C. Model predictive controller design

In [17], we presented a H_∞ based control approach for the control of heart rate response with exercises and obtained desired tracking performance for healthy young exercisers. In this study, we plan to cope with heart rate regulation for rehabilitation. In order to ensure the safety of exercisers, walking speed and acceleration of treadmill exercises must be confined in safety ranges. Model predictive control is the most suitable selection due to its intrinsic capability of dealing with constraints. After the pre-compensator is employed, the Hammerstein system can be treated as a linear dynamic system. Therefore, linear MPC can be applied to handle this problem.

Model predictive control predicts and optimize the future behaviour of the process based on a dynamic model of the process. At each control interval, the MPC algorithm calculates an open loop sequence of the manipulated variables in such a way to optimize the future behaviour of the plant [19]. The first value in this optimal sequence is injected into the plant. Figure 1 shows the state of a hypothetical SISO MPC system that has been operating for many sampling instants. Integer k represents the current instant. The latest measured output, y_k, and previous measurements, y_{k-1}, y_{k-2},..., are known.

To calculate its next move u_k, the controller operates in two phases [19]:

1. Estimation and Prediction: In order to make an intelligent move, the controller needs to know the current state and any internal variables that influence the future trend. To accomplish estimation and prediction, the controller uses all past and current measurements and the models.

2. Optimization: Values of setpoints, measured disturbances, and constraints are specified over a finite horizon of future sampling instants, $k+1$, $k+2$, $\cdots$, $k+p$, where p is the prediction horizon. The controller computes m moves u_k, u_{k+1}, ... u_{k+m-1}, where m is the control horizon. The moves are the solution of a constrained optimization problem:

$$\min_{\Delta u_k \cdots \Delta u_{k+m-1}} \Big(\sum_{l=1}^{p} \|\hat{y}_{k+l/k} - r_{k+l}\|^2_{\Gamma_l^y} + \sum_{l=1}^{m} \|\Delta u_{k+l-1}\|^2_{\Gamma_l^u}\Big), \tag{1}$$

where,

$\hat{y}_{k+l/k}$ is the predicted values of y at time $k+l$ based on information available at time k.

p is prediction horizon which sets the number of control intervals over which the controller predicts its outputs when computing controller moves.
m is control horizon which sets the number of moves computed. It must not exceed the prediction horizon. If less than the prediction horizon, the final computed move fills the remainder of the prediction horizon.
$\Delta u_k = u_k - u_{k-1}$.
$\|x\|^2_\Gamma = x^T \Gamma x$.
Γ^y_l and Γ^u_l are weighting matrix for predicted errors and control moves ($\Gamma^y_l > 0$ and $\Gamma^u_l \geq 0$). For SISO systems, Γ^y_l and Γ^u_l are nonnegtive scalars.

For details of the formulation, see [19] and [20].

In this paper, step response models obtained by using correlation analysis are applied to implement prediction. FIR type models (step response models and impulse response models) are the most common models utilised in commercial MPC packages [20]. The main reason is FIR model based predictions depend only on the input information, as these models have no auto-regressive part.

For a step response model, the following form is applied for prediction [20] [19]:

$$\hat{y}_{k+l/k} = \sum_{i=1}^{n-1} h_i \Delta u_{k+l-i} + h_n u_{k+l-n} + \hat{d}_{k+l|k}, \quad (2)$$

where
h_i $(i = 1, \cdots n)$ is the model step response matrix coefficient. n is the truncation order.
$\hat{d}_{k+l|k}$ is the predicted value of additive disturbance at process output at time $k + l$ based on information available at time k:

$$\hat{d}_{k+l|k} = y_m(k) - \sum_{i=1}^{n-1} h_i \Delta u_{k-i} + h_n u_{k+l-n}. \quad (3)$$

III. Experiment based modelling and simulation

A. Experimental equipment

The treadmill used in the system is the Powerjog "G" Series fully motorized medical grade treadmill manufactured by Sport Engineering Limited, England. Control of the treadmill can be achieved through an RS232 serial port. The treadmill can receive commands from the computer controller via this link, and obeys such commands without supervision. The measurement of heart rate in the designed system is implemented using a wireless Polar system.

B. Nonlinearity modelling by using Support Vector Regression

In order to identify the nonlinear relationship, steady state experiments are performed and recorded. Six young healthy male subjects volunteered to participate in the study. All experiments were conducted in the afternoon, and the subjects were permitted to have a light meal one hour before experiments. Initially, the subjects were asked to walk for about 10 minutes on the treadmill to familiarize themselves with the experiment. The subjects were then requested to walk at five levels of different speeds (3 km/h, 4 km/h, 5 km/h, 6 km/h and a subject specific maximum walking speed, typically 7km/hour). Each level took a total period of 5 minutes, and was followed by a 10-minute resting period. Finally, in order to identify the linear dynamic part of the Hammerstein system, subjects were also requested to walk on the treadmill under a PRBS input.

Kernel	Parameter	Regularization Constant C
RBF	σ=20.2	5
ϵ-insensitivity	Support vector number	Mean Square Error
0.8 km/h	5 (16.7 %)	0.26 (km/h)^2

TABLE I
Details about the estimation of the inverse of the nonlinear function by SVR

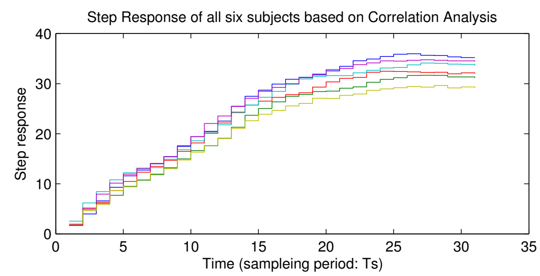

Fig. 2. Correlation analysis results of six subjects

In this paper, ϵ-insensitivity SVR regression methods is applied to model the inverse of the nonlinear function (f). The detailed regression results, such as the selected design parameters (including insensitivity region ϵ, kernel function, and the regularization constant C) and the support vector number, are summarized in Table I.

C. Linear dynamic modelling

For the dynamic modelling of heart rate variation during exercise, some complicated parametric models [21] [22] are proposed based on physiological analysis. However, because of the complexity of human body and the lacking of pre-knowledge, sometimes it is not proper to describe the response of human cardiovascular (includes heart rate response) to exercise by a fixed model structure. This paper applies a nonparametric modelling approach, called correlation analysis [23], to model the linear dynamic part of heart rate response. A PRBS [24] [25] type signal is one of the most suitable experimental signal for correlation analysis.

A PRBS input has two levels ($u(t) = u_1$ or u_2) and may switch from one level to the other only at constant time intervals T_s. A PRBS is periodic with period $T = T_s N$, where N is an integer. In this study, $u_1 = 4km/h, u_2 = 6km/h, N = 31$, and $T_s = 15s$.

Based on experimental data, correlation analysis is performed. The identified step response for six subjects are shown in Fig 2.

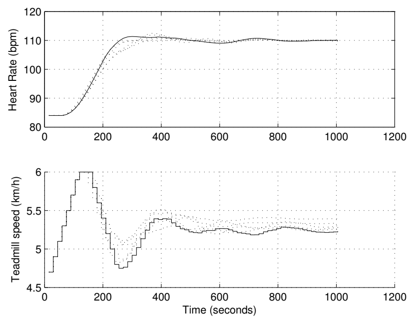

Fig. 3. Simulation results for all six subjects

D. MPC controller design for the regulation of heart rate

Before control system implementation, simulation studies are necessary for the investigation of the proposed MPC control approach. We mainly focus on the definition of system constraints, the selection of prediction and control horizons.

The simulation studies are performed for all six models identified by using the proposed Hammerstein model identification approach. As the inverse of the static nonlinearity has been identified and applied as a precompensator, the compensated system can be regarded as a linear dynamic system (described by a step response model). Then, the optimization problem associated with the MPC controller design can be described as in (1). The constraints of this optimization problem are defined as follows:

$$2_{(km/h)} \leq u \leq 6.5_{(km/h)};\ -0.2_{(km/h/T_s)} \leq \Delta u \leq 0.2_{(km/h/T_s)}. \quad (4)$$

There are no specific rules for the selection of prediction and control horizon. However, increasing p often results in less aggressive control action. Increasing m makes the controller more aggressive and increases computational effort. After extensive simulation studies, the best value of prediction horizon p and control horizon m are selects as is 35 and 5 respectively. Step responses for all six subjects (See Figure 3) demonstrate the effectiveness of the designed controller.

IV. Conclusion

One of the main purpose of the developed strategy is to achieve safer exercise in cardiac rehabilitation programs under actuator malfunctions and/or failures. Nonparametric Hammerstein model is established for heart rate response for treadmill exercises based on support vector machine regression and correlation analysis. Based on this nonparametric model, a MPC strategy is developed. In order to guarantee the safety of treadmill exercise for rehabilitators, the designed automated treadmill system is capable of optimizing system performance under predefined speed and acceleration range. Controller parameters, such as prediction and control horizons, have been predetermined based on simulation. We will apply the proposed approach in an experimental setting, and believe that its effectiveness will then be fully demonstrated.

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