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Deformation Modeling of Viscoelastic Objects for Their Shape Control

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Abstract

A new approach to the deformation modeling of viscoelastic objects for their shape control is presented. Manipulative operations of viscoelastic objects can be found in many industrial fields such as food industry and medical product industry. Automatic operations of viscoelastic objects are eagerly required in these fields. Since viscoelastic objects deform during operation processes, it is necessary to simulate the behavior of the objects and to estimate their deformation for the automatic operations. Consequently, a model of a viscoelastic object is needed for the simulation and the estimation of its deformation.

We will propose a lattice structure based modeling method for viscoelastic object deformation. First, behavior of four element models is briefly explained. Second, a viscoelastic object is modeled as a lattice structure, where mass points are connected through four element models. We will simulate shape deformation of the model when force input is applied to it. Validity of the model is then discussed. Next, we will introduce a nonlinear damper(NLD) into a four element model in order to solve a discrepancy between an actual viscoelastic object and its linear model. Comparing the behavior of the two models, we will show the validity of the model using NLD's.

1 Introduction

Manipulative operations of viscoelastic objects can be found in many fields. There exist forming operations of bread dough and pizza dough in food industry and handling operations of soft supplies in medical industry. Most handling operations of viscoelastic objects with large deformation depend upon humans. For example, the forming process of pizza dough includes a forming operation of the dough stretched by a roller. This operation is performed by humans and automatic forming of the stretched dough is required to improve product quality. Molding has been utilized in the automatic shape control of viscoelastic objects. However, we find many operations where the molding cannot be applied in food industry and in medical product industry. For example, the pizza dough must be shaped by extending the dough to ensure its quality. Molding the dough decreases the palate of a pizza. Thus, we have to develop new automatic machines for these operations without molding, and have to derive control strategies for the shape control of viscoelastic objects. Since the objects deform during operation processes, it is necessary to simulate the behavior of the objects and to estimate their deformation for the automatic operations. Consequently, a model of a viscoelastic object is needed for the simulation and the estimation of their deformation.

Modeling of viscoelastic objects has been studied in computer graphics and virtual reality. Terzopoulos et al. have proposed a method to the modeling of viscoelastic objects in computer graphics [1]. Joukhader et al. have proposed a modeling technique for deformable objects and have simulated the collision between deformable objects [2]. Chai et al. have developed a virtual reality system where users can deform virtual objects in computer [3]. These researches focus on deformed shape of the objects and their shaping process is out of consideration. Handling operations of deformable objects have been studied recently. Taylor et al. have experimentally studied automatic handling of deformable parts in garment industry and in shoe industry [4]. Zheng and Chen have proposed a strategy to insert a deformable beam into a hole [5]. Wada et al. have propose a control law for the positioning operation of extensible clothes [6]. These studies mainly deal with elastic objects. Shaping operation of viscoelastic objects are not studied.

In this paper, we will propose a lattice structure based modeling method for viscoelastic object deformation. First, behavior of four element models is briefly explained. Second, a viscoelastic object is modeled as a lattice structure, where mass points are connected through four element models. We will simulate shape deformation of the model when force input is applied to it. Validity of the model is then discussed. Next, we will introduce a nonlinear damper(NLD) into a four element model in order to solve a discrepancy between an actual viscoelastic object and its linear model. Comparing the behavior of the two models, we will show the validity of the model using NLD's.

2 Modeling of viscoelastic materials

In this section, we will briefly explain four element models, which describe viscoelastic nature of object materials. Elements describing viscoelastic materials are shown in Figure 1. Element shown in Figure 1-(a) is called Voigt model, that in Figure 1-(b) is called Maxwell model, and that in Figure 1-(c) is called four element model. Voigt model consists of a spring and a damper, which connect two mass points parallel. Maxwell model is a series of a spring and a damper connecting two mass points. Four element model is a series of a Voigt model and a Maxwell model, as shown in Figure 1-(c).

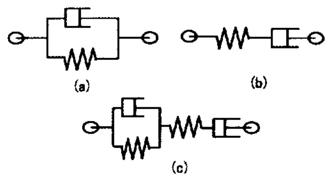

Figure 1: Viscoelastic elements: (a) Voigt model, (b) Maxwell model, (c) Four element model

Characteristics of the three models are described in Figure 2. Force of 1 [N] is applied to individual models during 10 seconds, as shown in Figure 2-(a). Responses to this force of individual models are plotted in Figure 2-(b). Graphs in this figure show deformation of individual models. Deformation of a Voigt model converges to an equilibrium when a constant force is applied to it. The deformation decreases and converges to zero after an applied force vanishes, as shown in Figure 2-(b). Namely, a Voigt model is capable of describing elasticity of materials. Deformation of a Maxwell model increases while a constant force is applied to it. The deformation reaches to a certain non-zero value after an applied force vanishes, as shown in Figure 2-(b). Namely, a Maxwell model can describe viscosity of materials. Deformation of a four element model is given by a sum of deformation of a Voigt part in the model and that of a Maxwell part. Thus, a four element model shows both characteristics of a Voigt model and that of a Maxwell model. In other words, four element model can describe viscoelastic nature of materials. From the above observation, we will use four element models to build a model of a viscoelastic object.

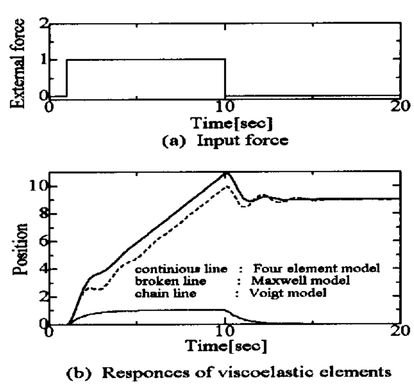

Figure 2: Characteristics of viscoelastic elements: (a) force input, (b) responses of viscoelastic elements

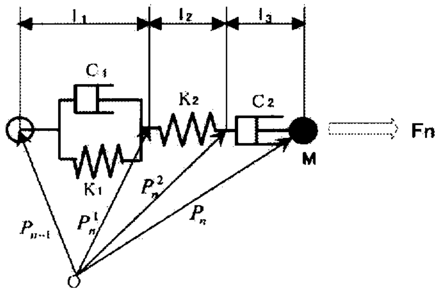

Figure 3: Four element model

Let us formulate the behavior of a four element model shown in Figure 3. Let O be the origin of a spatial coordinate system. Let $\boldsymbol{P}_{n-1}$ and $\boldsymbol{P}_n$ be coordinates of end points of a four element model. Recall that a four element model consists of a Voigt part and a Maxwell part. Spring constant and damper constant of a Voigt part in the model are denoted K_1 and C_1, respectively. Spring constant and damper constant of a Maxwell part in the model are denoted K_2 and C_2, respectively. Natural length of a Voigt part is given by l_1. Natural length of a spring of a Maxwell part is l_2 and that of a damper of a Maxwell part is l_3. Let M be mass at each end point. Let $\boldsymbol{P}_n^1$ be position of a point connecting a Voigt part and a Maxwell part, as described in Figure 3. Let $\boldsymbol{P}_n^2$ be position of a point connecting a spring and a damper in a Maxwell part, as illustrated in the figure. Note that points $\boldsymbol{P}_{n-1}$, $\boldsymbol{P}_n^1$, $\boldsymbol{P}_n^2$, and $\boldsymbol{P}_n$ exist on a straight line. Thus, position $\boldsymbol{P}_n^1$ and $\boldsymbol{P}_n^2$ can be described using parameters t_1 and t_2 as follows:

$$\begin{aligned}\boldsymbol{P}_n^1 &= t_1(\boldsymbol{P}_n - \boldsymbol{P}_{n-1}) + \boldsymbol{P}_{n-1} \\ \boldsymbol{P}_n^2 &= t_2(\boldsymbol{P}_n - \boldsymbol{P}_{n-1}) + \boldsymbol{P}_{n-1}\end{aligned}$$

Let $\boldsymbol{F}_e$ be a force applied to a mass point $\boldsymbol{P}_n$ by a four element model. Force $\boldsymbol{F}_e$ is equal to a force by the Voigt part. This yields

$$\begin{aligned}\boldsymbol{F}_e &= -C_1(t_1\dot{|\boldsymbol{P}_n - \boldsymbol{P}_{n-1}|})\frac{\boldsymbol{P}_n - \boldsymbol{P}_{n-1}}{|\boldsymbol{P}_n - \boldsymbol{P}_{n-1}|} \\ &\quad -K_1(t_1|\boldsymbol{P}_n - \boldsymbol{P}_{n-1}| - l_1)\frac{\boldsymbol{P}_n - \boldsymbol{P}_{n-1}}{|\boldsymbol{P}_n - \boldsymbol{P}_{n-1}|} \quad (1)\end{aligned}$$

Force $\boldsymbol{F}_e$ coincides a force by a spring in the Maxwell part. Thus, we have

$$\boldsymbol{F}_e = -K_2((t_2 - t_1)|\boldsymbol{P}_n - \boldsymbol{P}_{n-1}| - l_2)\frac{\boldsymbol{P}_n - \boldsymbol{P}_{n-1}}{|\boldsymbol{P}_n - \boldsymbol{P}_{n-1}|} \quad (2)$$

Force $\boldsymbol{F}_e$ is equal to a force by a damper in the Maxwell part. Namely,

$$\boldsymbol{F}_e = -C_2((1 - t_2)\dot{|\boldsymbol{P}_n - \boldsymbol{P}_{n-1}|})\frac{\boldsymbol{P}_n - \boldsymbol{P}_{n-1}}{|\boldsymbol{P}_n - \boldsymbol{P}_{n-1}|} \quad (3)$$

Let $\boldsymbol{F}_a$ be an external force acting on a mass point $\boldsymbol{P}_n$. Equation of a motion of a mass point at position

$\boldsymbol{P}_n$ is then given by

$$M\ddot{\boldsymbol{P}}_n = \boldsymbol{F}_e + \boldsymbol{F}_a \quad (4)$$

Equations (1) through (4) provide equation of a motion of a four element model. Eliminating vector $\boldsymbol{F}_e$ from equation (1), (2), and (3), we can determine parameters t_1 and t_2 uniquely. Substituting the values of t_1 and t_2 into one equation of (1), (2), and (3), we can determine the value of vector $\boldsymbol{F}_e$. Namely, we can compute a force by a four element model and can compute the motion of mass point $\boldsymbol{P}_n$.

3 Lattice model of viscoelastic objects

Viscoelastic objects deform in 3D space. Thus, we have to describe their deformation in 3D space. In this paper, we will propose a lattice structure to describe deformation of a viscoelastic object in 3D space. Let us distribute mass points in a natural shape of a viscoelastic object at the same intervals along x, y, and z-axis, as illustrated in Figure 4. Let N be the number of the mass points and M_{object} be the mass of the object. Then, mass of a point is given by $M = M_{object}/N$.

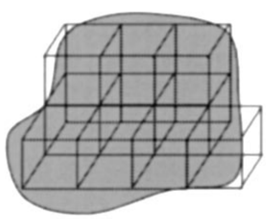

Figure 4: Lattice model of object

Four element models are inserted between all neighboring mass points, as illustrated in Figure 5. Namely, four element models are arranged along longitude, transverse, and diagonal directions. Viscoelastic deformation of an object can be described by deformation of these four element models. Let $\boldsymbol{P}_{i,j,k}$ be position vector corresponding to lattice point (i, j, k). Let us derive equation of motion of a mass point at $\boldsymbol{P}_{i,j,k}$. Force acting on $\boldsymbol{P}_{i,j,k}$ by a four element model between $\boldsymbol{P}_{i,j,k}$ and a neighboring point $\boldsymbol{P}_{i+\alpha,j+\beta,k+\gamma}$ is denoted by $\boldsymbol{F}^{\alpha,\beta,\gamma}_{i,j,k}$. Then, total internal force acting on $\boldsymbol{P}_{i,j,k}$ is given by the sum of $\boldsymbol{F}^{\alpha,\beta,\gamma}_{i,j,k}$, that is,

$$\boldsymbol{F}^{e}_{i,j,k} = \sum_{\substack{\alpha,\beta,\gamma\in\{-1,0,1\} \\ (\alpha,\beta,\gamma)\neq(0,0,0)}} \boldsymbol{F}^{\alpha,\beta,\gamma}_{i,j,k}$$

Recall that force $\boldsymbol{F}^{\alpha,\beta,\gamma}_{i,j,k}$ can be computed using a procedure explained in section 2. Thus, force $\boldsymbol{F}^{e}_{i,j,k}$ can be computed by summing all forces. Let $\boldsymbol{F}^{a}_{i,j,k}$ be total external force acting on $\boldsymbol{P}_{i,j,k}$. The equation of motion is thus described as follows:

$$M\ddot{\boldsymbol{P}}_{i,j,k} = \boldsymbol{F}^{e}_{i,j,k} + \boldsymbol{F}^{a}_{i,j,k}$$

By solving a set of equations corresponding to all mass points consisting of the model, we can compute the deformation of a viscoelastic object.

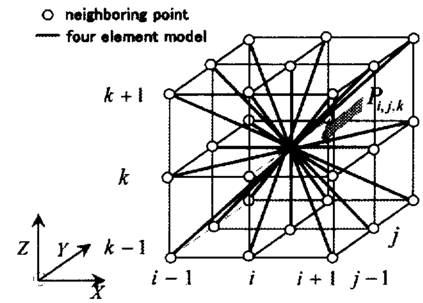

Figure 5: Neighboring lattice points and four element models

4 Simulation results of 2D model

In this section, we will show simulation results of deformation of a viscoelastic object using its two-dimensional model. Computation process described in Section 3 can be applied to two-dimensional models. In addition, computation results using two-dimensional models can easily extended to 3D deformation of viscoelastic objects.

Let us consider a lattice model for a cubic object shown in Figure 6. This lattice model consists of 6×6 mass points. Four element models are inserted between all neighboring mass points. 25 elements along x-axis, 25 elements along y-axis, and 50 diagonal elements are involved in the model.

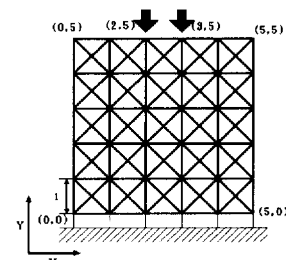

Figure 6: Two-dimensional model of cubic object

Let us compute deformation of the model corresponding to force input at its surface. Mass of all mass points is assumed to be $M = 0.1$. Values of spring constants and damper constants in all four element models are assumed as $K_1 = K_2 = 1.0$ and $C_1 = C_2 = 1.0$. The bottom of the object is fixed to the space.

Force input: Let us impose force input on the model at points corresponding to (2,5) and (3,5). Step force input at the two points is given in Figure 7-(a). Step force of 0.2 along y-axis in the negative direction is applied to the two points for 2 seconds. Figure 7-(b) shows the initial shape and a deformed shape in the stationary condition.

From the simulation results shown in Figure 7-(a) and (b), we find that the deformation of a viscoelastic object can be computed and the proposed model shows a behavior similar to an actual viscoelastic object. But one discrepancy between that of the model and that of an actual object is found. Assume that a small force is applied to a viscoelastic object. For example, imagine a situation that a penny is located on an amount of bread dough. Then, the bread dough may not deform in a short time. On the other side, the proposed model deforms in a short time even if a small force is exerted on it. Figure 7-(c) and (d) shows the deformation of the model for a small force. Figure 7-(c) describes step force input of 0.01 applied to two points corresponding to (2,5) and (3,5). Deformed shape after 50 seconds is described in Figure 7-(d). Note that the object continues to deform while a small force is applied to it. Namely, the deformation does not converge. This discrepancy between the model and an actual object is due to linearity of a four element model. In the next section, we will introduce a nonlinear four element model to solve this discrepancy.

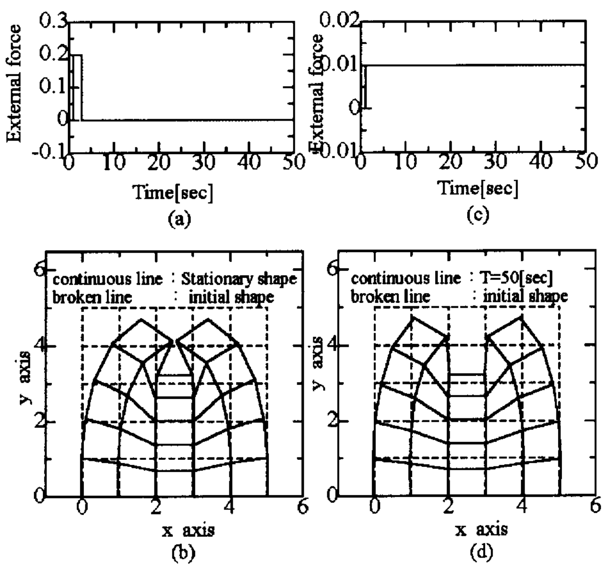

Figure 7: Response of two-dimensional model for force input: (a) force input at points (2,5) and (3,5)(F=0.2), (b) deformed shape in stationary condition by force of (a), (c) force input at points (2,5) and (3,5)(F=0.01), (d) deformed shape after 50 seconds by force of (c)

5 Four element model using nonlinear damper

As discussed in the previous section, a viscoelastic object may not deform for a small force. In this section, we will introduce a nonlinear four element model to describe this behavior of a viscoelastic object. Discrepancy discussed in the previous section is caused by a linear damper in the Maxwell part of a four element model. This damper continues to extend while a small force is applied to the damper. To solve the discrepancy, the damper should not extend for a small-applied force while it can behave as a linear damper otherwise. Thus, we will introduce a nonlinear damper (NLD) into a four element model. Damper constant of the NLD should be large for a small force while should be an appropriate constant value otherwise. Namely, damper constant of the NLD depends on a force applied to the NLD. Let f be the magnitude of a force applied to an NLD and $C_2(f)$ be damper coefficient of the NLD. Function $C_2(f)$ must be a monotonously decreasing function. For example, we can define function $C_2(f)$ as follows:

$$C_2(f) = \begin{cases} C_{MAX} & (F < F_0 + \epsilon) \\ \dfrac{Acot^{-1}(B(F - F_0))}{F - F_0} + C_{MIN} & (F \geq F_0 + \epsilon) \end{cases}$$

where A, B and ϵ are constants. Maximum value and minimum value of the damper coefficient are given by C_{MAX} and C_{MIN}, respectively. Value of C_{MAX} must be large enough. Then, the NLD does not extend when the applied force is smaller than $F_0+\epsilon$. The NLD extends when force exceeds $F_0 + \epsilon$. Thus, parameter F_0 defines a limit whether the NLD extends or not. An example of function $C_2(f)$ is plotted in Figure 10. In this example, $A = 10$, $B = 10$, $C = 1$, $\epsilon = 10^{-8}$, $C_{MAX} = 1.6 \times 10^9$, and $F_0 = 1$. As shown in the figure, value of $C_2(f)$ is large around $f = 0$ while $C_2(f)$ has a constant value when f is greater than $F_0 + \epsilon = 1 + 10^{-8}$.

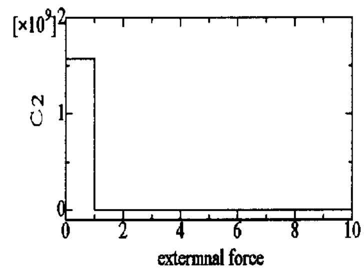

Figure 8: Coefficient of nonlinear damper

Figure 9 shows response to step input of force on a normal four element model and that on a four element model using an NLD. Broken lines in the figures describe the response of a normal element while continuous lines show the response of an element using an NLD. The magnitude of force F is 1 in Figure 9-(a) and is 0.1 in Figure 9-(b). As shown in Figure 9-(a), response of a four element model using an NLD is almost same as that of a normal element when $F = 0.1$. On the other hand, a normal four element continues to deform while deformation of a four element model using an NLD converges to a certain value when $F = 1$, as plotted in Figure 9-(b). Recall that damper coefficient $C_2(f)$ of the NLD has a large value when F is around 0.1. This yields the response shown in the figure.

As discussed above, response of a four element model using an NLD is similar to that of an actual viscoelastic object. We will therefore use the nonlinear model to build models of viscoelastic objects.

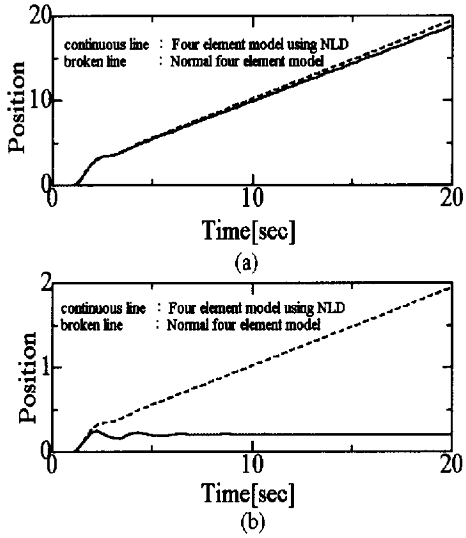

Figure 9: Response of four element model using NLD for step input of force: (a) external force $F = 1$, (b) external force $F = 0.1$

6 Simulation results of deformation using nonlinear four element models

In this section, we will show simulation results of object deformation using nonlinear four element models. Let us build a two-dimensional lattice model of a viscoelastic object using four element models with NLD's. The model consists of 6×6 mass points to describe a cubic shape. Deformation for force input and that for displacement input are computed here.

Force input: Let us impose force input to the surface of the model at the two points. Step force input at the two points is given in Figure 10-(a). Step force of 0.2 along y-axis in the negative direction is applied to the two points for 2 seconds as plotted in Figure 7-(a). Figure 10-(b) describes the initial shape of the model and the deformed shape in stationary condition. Comparing Figure 10-(b) with Figure 7-(b), we find that the model using NLD's deforms as the normal model does. Figure 10-(c) shows step force of 0.01 applied to the two points. Figure 10-(d) describes the initial shape of the model and the deformed shape in stationary condition. Recall that the linear model continues to deform for any small force, as shown in Figure 7-(d). Comparing Figure 10-(d) with Figure 7-(d), we find that the model using NLD's does not deform for a relatively small force. Consequently, introducing a nonlinear damper to a four element model enables us to build a model more similar to an actual viscoelastic object.

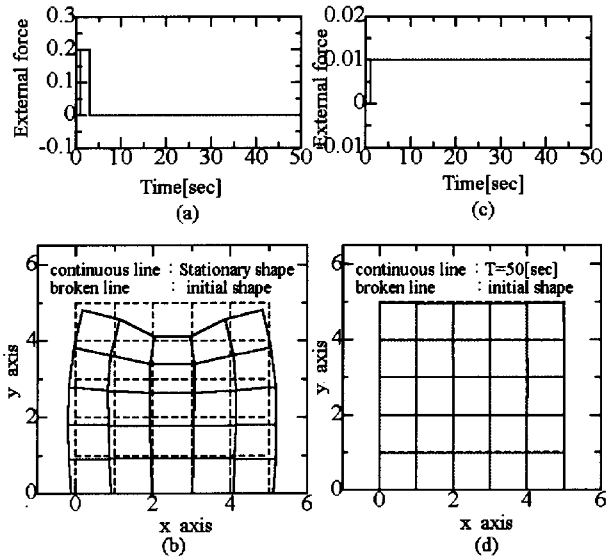

Figure 10: Response of two-dimensional model using NLD's for force input: (a) force input at lattice points (2,5) and (3,5), (b) deformed shape in stationary condition

Displacement input: Let us impose displacement input to the surface of the model at two lattice points (2,5) and (3,5). Displacement input at the two points is given in Figure 11-(a). Value of y-coordinate at the two points decreases from 5.0 to 3.5 during 2 seconds. Figure 11-(b) describes an initial shape of the model and its deformed shape after 3.6 seconds. Figure 11-(c) shows the initial shape and a deformed shape in the stationary condition.

Comparing these figures, we find that both topsides of the model deform toward the center of the model, where displacement input is applied. Moreover, we find that the model expands along x-axis.

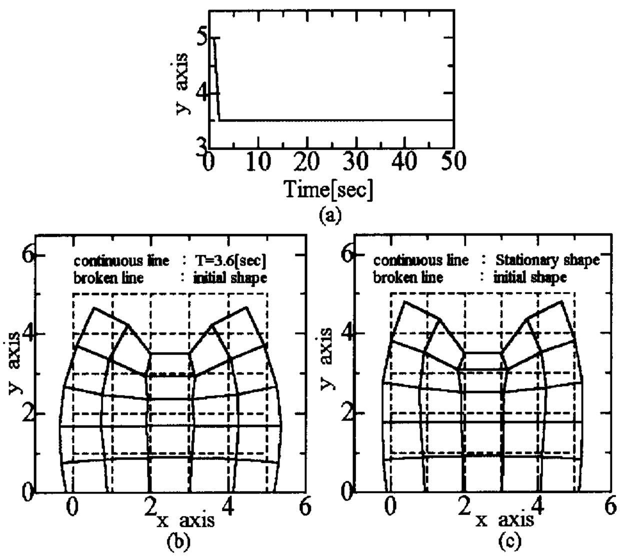

Figure 11: Response of two-dimensional model using NLD for displacement input: (a) position input at lattice points (2,5) and (3,5), (b) deformed shape after 3.6 seconds, (c) deformed shape in stationary condition

7 Simulation of shaping process

In this section, the shaping process of a viscoelastic object is simulated. A viscoelastic object is shaped with a rotating roller, as illustrated in Figure 12.

In this simulation, a viscoelastic object is modeled by the lattice structure of four element models described in Section 4. Radius of the roller is assumed to 1. The roller is assumed to move from the initial position in the direction of x at a speed of 0.4. The roller rotates by 1(rad/sec). This simulation result is shown in Figure 13.

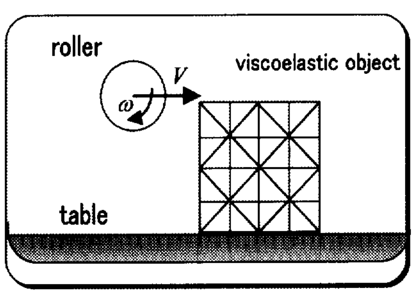

Figure 12: Shaping of viscoelastic object by roller

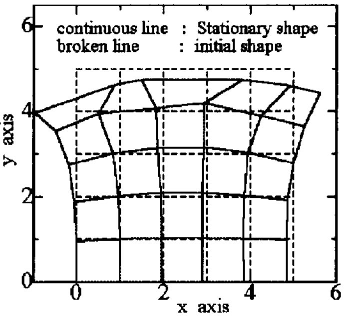

Figure 13: Response of two-dimensional model in shaping by roller

8 Conclusion

In this paper, we have proposed a new method to the modeling of viscoelastic objects for their deformation control. We have proposed a lattice structure to describe the deformation of a viscoelastic object. Nonlinear dampers are introduced to the object model so that the deformation corresponding to small forces can be described appropriately.

Future problems are (1) identification of parameters in four element models, (2) experimental verification of the proposed approach, and (3) derivation of control strategies for shape control of viscoelastic objects.

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