Energy Efficient Topology for Wireless Microsensor Networks

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ABSTRACT

We analyze the relationship between energy consumption and topology in wireless microsensor networks. Energy consumption is the total energy required for a message to be delivered to a destination microsensor from a source microsensor. We first consider the factors in energy consumption – radio propagation models, the topology of microsensors, the probability of connectivity between microsensors and etc. The radio propagation model tells us how much energy the environment of message propagation requires, and the topology is the logical configuration among microsensors. We analyze the energy consumption from two different aspects: the propagation model and the topology. A propagation model may consume more energy than that of another model at the same topology, or, different topologies may require different amounts of energy in the same propagation model. The result of analysis shows that the consumed energy is in proportion to the number of neighbors, i.e. when the topology has fewer neighboring microsensors, it consumes less energy even though it must experience more hops to the destination. We also prove that the same result can be applied to any of the radio propagation models – such as free space propagation, urban area, obstructed in building, and etc. From the two analyses in the propagation model and topology, we can conclude that when a message goes to the destination of multihop, the topology with fewer neighboring microsensors consumes less energy than that of more neighbors. We also perform a simple analysis on the connectivity among microsensors as one of the energy consumption factors. Microsensors are prone to be disconnected by microsensor failures, temporary broken links, going into sleep mode, and etc. The disconnection requires an alternative path to the destination and (or) retransmission of the same message to the next microssensor, which consumes additional energy.

Categories and Subject Descriptors

C.2.1 [**Computer-Communication Networks**]: Network Architecture and Design – *wireless communication, network topology*; C.2.3 [**Computer-Communication Networks**]: Network Operations – *network management*.

General Terms

Theory, Management, Reliability.

Keywords

Minimum Energy, Topology, Sensor Networks.

1. PROBLEM

Recent advances in sensor technology have resulted in the availability of inexpensive wireless microsensors [5], [9]. Although these microsensors are not as reliable, durable or accurate as existing deployed sensors, it is possible to achieve a desired result (reliability, accuracy etc.) via the networking of microsensors [13], [24]. The deployment of networked microsensors, i.e. topology, must be energy efficient (e.g. minimize energy consumption to the destination) due to the limited energy of microsensors. Therefore, the following question is raised:

> " Given a set of networked microsensors in the same area with the same number of wireless microsensors, which topology incurs the minimum energy to go to the destination?"

Figure 1 shows one of deployments, i.e. topology, with the same number of microsensors and network area. The bullets represent microsensors and the dotted circle is the transmission range by the centered microsensor. The lines among bullets are communication links. Microsensor A has four (left) and six neighbors (right). Assume that the distance from S to D is the same[1] in both sides of Figure 1. To go to destination D from S, there can be many decisions, i.e. how many neighboring microsensors exist in one hop propagation. The more neighbors a microsensor has, the larger the microsensor's transmission range is. If we have a larger transmission range, we experience a smaller number of hops to the destination and vice versa (see Section 4). One way to answer the above question is to find the relationship between topology and energy consumption. Topology is determined by the number of neighboring microsensors in the area covered by the transmission range of a microsensor (e.g., the dotted circle in Figure 1).

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[1] The distance from S to D in the right side is longer than that of the left.

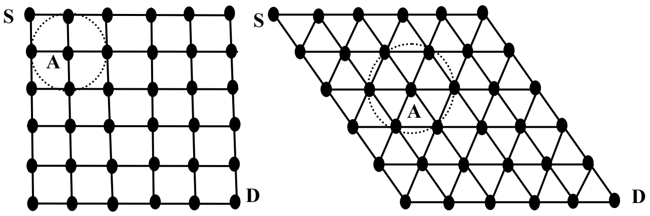

Figure 1. Two topologies of four and six neighbors.

The larger the transmission range of a microsensor is, the more neighbors the microsensor has. The more neighbors a microsensor has, the more energy it consumes. A larger transmission range can be prepared using larger transmission energy [19]. A microsensor network with a larger transmission range also consumes more energy than that with a smaller transmission range because the larger number of neighbors requires more energy to receive messages.

In this paper, we analyze the relationship between energy consumption and topology. The analysis proves that the consumed energy is in proportion to the number of neighbors; i.e. when the topology has fewer neighboring microsensors, it consumes less energy even though it has to experience more hops to the destination. We perform another analysis on the connectivity among microsensors as one of the energy consumption factors. Disconnections to the destination require searches of alternative paths and (or) retransmission of the same message to the next microsensor, which consumes additional energy. In Section 2, we describe related work. Section 3 represents the factors of energy consumption and the connectivity analysis in Section 4. In Section 5, the energy consumption analysis includes three analyses from different aspects: the propagation model, the topology, and the connection probability. Section 6 discusses very briefly the energy required to react the network partition. Our conclusions and future work are presented in Section 7.

2. RELATED WORK

Many researches have been focused on energy efficiency in various aspects but not on the relationship between energy consumption and topology. For instance, communication protocols focus on 1) optimizing the number of hops [14], [15]; 2) the number of communication messages [3], [22], [14], [15]; 3) the minimization of transmission energy [3], [22]; and 4) load balancing by dynamic assignment of cluster heads [18]. The consumed energy for transmitting a message from a source to a destination can be reduced by dividing a distance into several shorter ones [6]. This approach is called a multihop. However, the shorter distance causes another problem – more relays to the same destination, which in turn dissipate more energy than the longer distance. Many multihop routing protocols [12], [7], [17] are referred to as energy-efficient since they adopt reduced transmission ranges and require a lower number of hops. Clearly they save energy on the one hand; but on the other hand, they have the potential to save much more energy through the consideration of the topology.

Topology management schemes such as SPAN [8] and GAF [25] have been proposed in multihop routing and both exploit network density for energy saving. In SPAN [8], a partial set of microsensors builds a virtual backbone and non-backbone microsensors go into sleep mode more frequently for energy saving. Backbone microsensors are replaced by non-backbone ones when their energy consumption reaches a threshold. Geographic Adaptive Fidelity (GAF) [25] divides a network into many small grids, where a number of microsensors exist and only one microsensor is active at a time, similar to a cluster head. Like the clustered approach, one of the inactive microsensors becomes active when the energy level of the current active one reaches a threshold. GAF saves energy significantly, but it needs dense networks. STEM [21] exploits both network density and packet latency. Though the existing approaches are energy efficient schemes, they do not deal with the relationship between energy consumption and topology. We believe that energy consumption highly depends on the topology in microsensor networks. However, to the authors' knowledge, there has been no research on the relationship between energy consumption and topology. A limited amount of energy also incurs unstable connections among microsensors, which require additional energy. We will analyze the topology and connectivity from the aspect of energy consumption in the sections that follow.

3. PRELIMINARIES

In this section, we describe a model for energy consumption and radio propagation. A microsensor network is modeled using a unit disk graph [10], in which all nodes (microsensors) have the same length of an edge (transmission range). The energy consumption model concurs with the real energy dissipation in a microsensor system [18], [6].

3.1 Wireless Microsensor Network Model

A wireless microsensor network consists of a finite set of nodes and a finite set of channels, the communication paths between nodes can be represented as a unit disk graph [10], G = (V, E), where there exists a connection between two nodes if they are within the transmission range of each other. A unit disk graph means that all nodes have the same transmission range. A delay for message transmission is arbitrary but finite. We assume that the locations of neighboring microsensors, i.e. microsensor locations within the transmission range of a given microsensor, are discovered and maintained in each microsensor. Since new locations due to mobility, failures and etc. can be achieved using periodic beacons, a more energy efficient location discovery algorithm is proposed [20]. The effect of energy consumption by location discovery can be ignored because all neighboring microsensors must execute the same location discovery algorithm, i.e. all neighbors consume the same amount of energy.

3.2 Factors of Energy Consumption

There are many factors to consider in energy consumption – topology, propagation model, message length, wireless wave length, antenna gains, propagation protocol, data rate, distance to the destination, microsensor's energy saving strategy, i.e. when and how many microsensors go into sleep mode, the total number of microsensors in a given network area (or the network area with a given number of microsensors), signal and noise ratio, movement speeds of microsensors and etc. Some factors are dependent on others, e.g., the size of a transmission range is affected by the total number of neighbors in a given network area.

Message length does not have a direct relationship to the other factors. The propagation model includes other factors such as wave length, signal noise ratio, data rate, and antenna gains [19], [16]. The transmission range can be decided using the number of microsensors and the network area (see Section 4). The amount of energy to process an incoming message for the selection of the next microsensor is negligible [11]. The movement of a microsensor does not affect other factors and can be treated as a connection or disconnection. If a microsensor moves into another location but is still within the transmission range, the movement does not affect other factors. Instead it only consumes its own energy to move, i.e. kinetic energy. Thus, for energy consumption in a given wireless microsensor network, it is enough to consider only topology, the propagation model and propagation protocols. In addition, connectivity among microsensors must be explored because disconnections or broken links require retransmissions and (or) alternative paths to the destination, which consume additional energy.

(1) Topology

The topology shows logical connections, i.e. message paths, among microsensors and the roles of microsensors in the logical connections. For instance, in a one-hop clustering configuration, logical connections are all links of one hop distance from a cluster head and the roles are cluster head or cluster member. However, when a message goes to the destination of multihop distance, energy consumption is only related to the number of neighbors, i.e. how many microsensors are involved in the receipt and retransmission of an incoming message. If the destination is within a one hop distance, energy consumption is still affected by the number of neighboring microsensors. The propagation protocol decides which retransmits and receives an incoming message. In wireless microsensor networks, a message propagates by one hop and total energy consumption is the summation of the consumed energy by one hop propagation of the message. Hence, the logical connections and the roles do not affect the energy consumption[2] and only the number of neighbors decides the amount of energy in one hop.

(2) Radio propagation model

The propagation model shows the prediction of an average received signal strength at a given distance from a transmitter. Radio propagation generally experiences reflection, diffraction, and scattering by various objects. Generally, received signal strength falls as $1/d^n$, where d is the distance between the transmitter and the receiver and n is called a path loss exponent [19]. Path loss exponents are not the same in different propagation environments: i.e. 2 in free space, 4 to 6 in obstructed in building, 2.7 to 3.5 in urban area cellular radio and etc. Intuitively when there are many candidate paths to the next microsensor, the minimum energy path is not the minimum distance to the next microsensor; a received signal decreases as $1/d^n$, a detour path, which is not the shortest path, may consume less energy. For instance, in the free space model (the path loss exponent is 2), the required energy to send a message to the next microsensor at distance d is in proportion to d^2 (assume that the energy consumption is related only to the distance). Assume a detour path has two one-hop distances, b and c, and a direct path has the distance of z. The energy consumption by the detour path is (b^2+c^2) and the direct path consumes z^2. When the distance of z is given, the values of b and c, which satisfy the condition of $\{(b^2+c^2) < z^2\}$ and $\{(b+c) > z\}$[3], can be easily found, e.g. b=1, c=1 and z=1.5. When the path loss exponent is different, i.e. the propagation environment is different, the total consumed energy to the destination is different.

(3) Propagation protocol

The propagation protocol selects both the path to the next microsensor and which microsensor receives and retransmits an incoming message. Consequently, the protocol decides the number of receivers and transmitters, which affects the amount of consumed energy. Three typical protocols are used in the analysis: only one neighbor retransmits, all neighbors retransmit, and average (k-1) neighbors retransmit. Though some protocols like directional propagation and source routing can reduce the amount of consumed energy, they require knowledge on a pre-determined path, i.e. know which microsensor is the next, and (or) the location of a destination. However, they may consume more energy than other protocols since connectivity among microsensors is very prone to break for reasons such as microsensor failure, an operation mode change among transmit/receive/idle/sleep mode, a temporary broken link or a movement to another location. When a next microsensor in a pre-defined path is not available, the protocol has to find a new candidate path to the destination. This also dissipates energy.

(4) Connectivity among microsensors

Wireless networks provide much lower connectivity among neighboring participants than wired networks [19], [1]. When participants are microsensors, i.e. microsensor networks, the limited energy of the microsensor incurs unstable connections to neighboring microsensors. This brings additional energy consumption. Connectivity is related to other network parameters like transmission range, network area, the total number of microsensors, and the number of neighboring microsensors. Connectivity is analyzed in detail in Section 4.

Energy efficient topology in microsensor networks, which incurs less consumed energy to the destination, must be considered collectively from the aspects of: 1) topology itself, 2) radio propagation model, 3) propagation protocol, and 4) connectivity among microsensors. The total consumed energy has to include the effects by 1) the number of messages, 2) the transmission range, 3) energy dissipation for the receipt and transmission of messages, and 4) distance from source to destination.

3.3 Energy Consumption Model for Microsensor Networks

Figure 2 represents the first order radio model [18], which shows energy consumption in each component of a microsensor to transmit and receive an s-bit message. $E_{Tx\text{-}elec}$ and $E_{Rx\text{-}elec}$ are the energy of the transmitter and the receiver electronics per bit and the same as E_{elec} in all the given microsensors. The required energy per bit for the transmit amplifier is represented as E_{amp}. Hence, necessary energy to send (receive) an s-bit message to (from) the node of the distance d is as follows:

$$E_{Tx}(d) = \{(E_{elec} * s) + (E_{amp} * s * d^2)\}. \quad (1)$$

$$E_{Rx} = (E_{elec} * s). \quad (2)$$

Energy consumption is in proportion to the distance between the transmitter and the receiver, and the radio characteristics of E_{elec} and E_{amp}. The radio characteristics are chosen from current radio

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[2] Actually the propagation protocol selects the next hop using the topology information; thus it is thought to affect the energy consumption to the destination of multihop distance indirectly.

[3] The condition is the property of triangles.

design, e.g. Bluetooth specification shows 115nJ/bit of E_{elec} [2]. The path loss in the free space propagation model is given as $-10\log[(G_tG_rw^2)/((4\pi)^2d^2)]$, i.e. the path loss exponent is 2, where G_t and G_r are the antenna gains of the transmitter and the receiver, w is the wavelength and d is the distance to the receiver [19]. When radio characteristics like G_t, G_r and w are given, the required energy by the transmitter (or the energy consumption by the distance d) can be simply written as a form of d^2. Hence, the first order radio model can be thought to work both in the free space propagation environment and between two microsensors of distance d. If microsensor networks are deployed in other propagation models like *obstructed in building*, where the path loss exponent is 4 to 6, the first order radio model can be applied by adjusting the distance d into g using the condition $d^{(4\text{ to }6)} = g^2$.

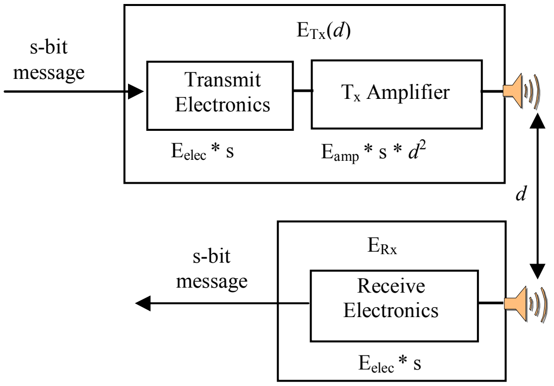

Figure 2. First order radio model.

4. CONNECTIVITY ANALYSIS

In this section, we first survey basic properties like minimum transmission range, the minimum number of microsensors to cover a given network area, and etc. With these properties, we compare connectivity, i.e. the probability to the destination between shorter and longer transmission ranges. The comparison shows that the shorter the transmission range of a microsensor is, the fewer neighbors it has. We also analyze the amount of neighboring microsensors needed to reach the destination with the desired probability and the amount of microsensors which we can expect from a given transmission range and network density, N/A. In an example of 400 microsensors deployed in a (10m x 10m) network area, we show that a simple connectivity analysis concurs with our expectation.

4.1 Basic Properties

We assume that all microsensors communicate with omni-directional neighbors, i.e. they do not adopt directional antennas. Let N be the total number of microsensors in the network area A. We also assume that r represents an arbitrary transmission range and k is the number of microsensors within the area covered by r. Network density is N/A.

Property 1 (Number of Microsensors)

The number of microsensors in an arbitrary network area is proportional to the transmission range r.

(Proof) When r is a given transmission range, the number of microsensors in the area is $(N/A) * \pi r^2$, simply k. When all variables except r are fixed, k is proportional to r. The larger r is, the larger k is.

Property 2 (Minimum Number of Microsensors)

Two neighbors are the minimum number of neighbors to pass a message to the microsensor of multihop distance.

(Proof) Except the source and the destination microsensor, every intermediary microsensor needs at least one sender and one receiver to relay a message.

Property 3 (Minimum Transmission Range)

The minimum transmission range for propagation in a given network is $\sqrt{(3/\pi) * (A/N)}$.

(Proof) The minimum number of microsensors for propagation is three, i.e. two neighbors and one as a message sender, and from property 1 we have the minimum transmission range, $\sqrt{(3/\pi) * (A/N)}$.

Property 4 (Minimum Number of Microsensors for Area Coverage)

To cover the given network area A with the transmission range r, the minimal number of microsensors should be satisfied by the equation, $\{A/(N * \pi r^2)\} = (2\pi)/\sqrt{27}$.

(Proof) [23].

4.2 Probability to the Destination

We assume that there are N microsensors randomly deployed in network area A and M is the number of microsensors in the area of transmission range r, i.e. $a = \pi r^2$. The probability that any microsensor is within area a is $p = \pi r^2/A$. Similarly the probability that there are k microsensors within area a is

$$P[M=k] = C_N^k\, p^k(1-p)^{N-k}. \tag{3}$$

If we fix r, N and A, $pN = (\pi r^2/A) * N$ is constant. When pN is constant, the equation of (3) has the Poisson distribution of the parameter, pN. Thus, equation (3) can be written as follows:

$$P[M=k] = \{(pN)^k e^{-(pN)}\}/k!,\ k = 0,1,.., N. \tag{4}$$

When a given network, i.e. r, N and A, is fixed, we can calculate the probability of a different number of neighbors. The number of neighbors is (k-1) and we can save unnecessary energy by forcing some neighboring microsensors to be in sleep mode, not allowing them to be involved in the message relay. Assume that the maximum probability of equation (4) occurs when the number of neighbors is five, i.e. $k = 6$, and the number of maximally allowed neighbors under the given r, N and A is 7. In such an environment, one neighboring microsensor is forced to be in sleep mode; thus, only six microsensors work in the transmission and relaying of messages. Microsensor networks do not guarantee a path to the destination even though they have multiple neighboring microsensors.

When a microsensor has (k-1) neighboring microsensors and has received the same requests (e.g. send the same message to the same destination) from q neighbors, it should designate the remaining neighbors as new relaying microsensors and transmit the message. But all the remaining (k-1- q) microsensors may be in sleep mode, be failures, or be too short of energy to relay a message. Hence, every microsensor should reserve α-connectivity, i.e. α-available candidate microsensors to the destination within the transmission range, to increase the probability of reachability to the destination. The α-connectivity can be easily determined from k, q, r, N, A, and the desired probability to the destination. For instance, if we want two times connectivity, i.e. from α to 2α, the transmission range should be extended by times, i.e. from r to 1.4r. If we do not want to change the transmission range for energy saving, the network density (say *c*) should be increased two times, i.e. from *c* to 2*c*. Similarly the network area, A, may be adjusted to half of A.

The probability that the number of neighboring microsensors is no less than a specific value *k* is calculated by

$$P[M \geq k] = 1 - P[M < k] = 1 - e^{-(pN)} \sum_{i=0}^{k-1}\{(pN)^i / i!\}\,. \quad (5)$$

From equation (5), we can estimate the maximum neighboring microsensors under a given probability. If we want at least five neighboring microsensors with 99% under the same network density of N/A, i.e. 400/100m^2, the minimum transmission range should be 0.8m. With a smaller transmission range than 0.8m, the network area should be changed into a smaller one, i.e. from A to *c*A, where $c < 1$ or more microsensors should be deployed into the same network area, i.e. from N to *c*N, where $c > 1$.

(Example 1)

Figure 3 shows 400 microsensors, which are randomly deployed in a network area of 100m^2, 10m x 10m. In Figure 4, the probability of different ranges depends on the number of neighbors, (*k*-1). The transmission range changes from 0.5m to 1.1m and the network density, i.e. N/A, is 4/m^2. When r is 0.6m, the expected number of microsensors becomes 4.5 from (πr^2) x (N/A), i.e. 3.5 neighboring microsensors, and the probability of (*k*-1) is the highest, i.e. 0.189, when (*k*-1) is 3. Figure 5 represents a probability of no less than (*k*-1) in the given parameters: r, N, and A. If we need *t*-resilient paths to the destination and an average of *q* microsensors from neighbors are not operational, the inequality of $(k - 1 - q - t) > 0$ should be satisfied.

5. ENERGY CONSUMPTION ANALYSIS

In this section we analyze the total energy consumption to the destination from two aspects: the radio propagation model and topology. From them, we can conclude that a topology with a smaller number of neighbors, i.e. a shorter transmission range, consumes less energy than that with a larger number of neighbors, i.e. a longer transmission range. Later we also explore the energy consumption by connectivity; that is, how much energy is required to go the destination under a given connection probability

5.1 Preliminaries

Transmission of a single message over a single communication link is a basic unit of energy and complexity analysis. The amount of necessary energy to send a message depends on message length and the distance between source and destination microsensors. We assume that the propagation protocol transmits the incoming message to its neighbors once. The received message first checks whether the incoming message has already reached its destination. If the receiver is the destination, there is no more work. If the receiver is not the destination, the receiver plays the role of an intermediary. The receiver relays the incoming message to the neighbors which do not send the same message to the receiver.

5.2 Energy Consumption in the Propagation Model

We also consider the radio propagation environments of the different path loss exponents, which consequently result in different energy consumption [19].

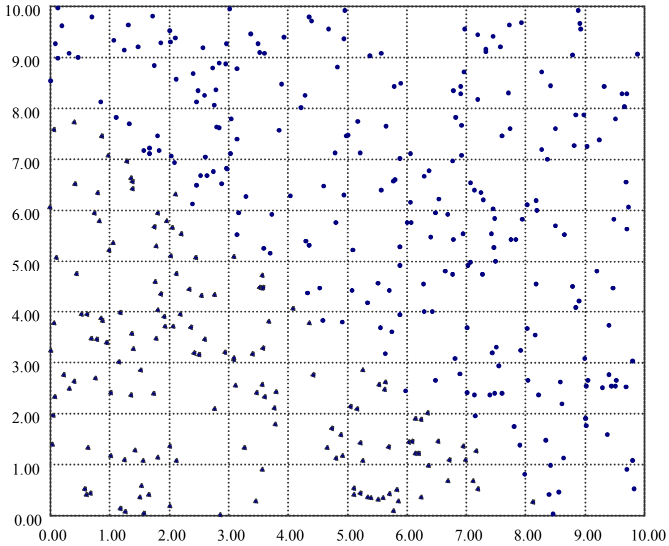

Figure 3. Randomly deployed microsensors.

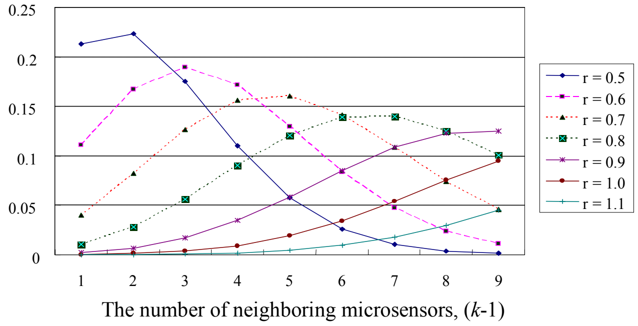

Figure 4. Probability of P[M=k] on transmission range r.

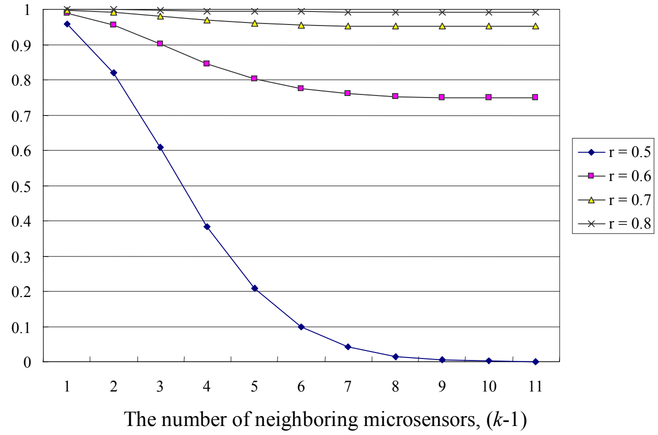

Figure 5. Probability of no less than (k-1).

In the free space propagation model, we prove that the shorter transmission range approach consumes less energy than that of the longer transmission range. The conclusion can also be applied to other propagation modes. Thus, the shorter the transmission range is, the less the total consumed energy is.

In propagation model based analysis, we assume that even though there are many neighboring microsensors, only one of them relays the message to the destination. If the protocol allows more than one microsensor to transmit the same message, the analysis result will be the same because the multiple transmissions are a repeated execution of a single message transmission.

Theorem 1 (Free Space Propagation Model)

The topology with two neighbors consumes the minimum amount of energy to the destination under given A, N, and *d*. We assume that the relationship among A, N, and r satisfies property 4.

(Proof)

From property 3, we know that the minimal transmission range is $\sqrt{(3/\pi)*(A/N)}$ and the number of hops, i.e. the number of message relay by microsensors is $(d/\sqrt{(3/\pi)*(A/N)})$.

The minimal transmission range occurs when there are two neighbors, i.e. three microsensors in the area by the transmission range r. Similarly with (k-1) neighbors, the number of hops is $\{d/\sqrt{(k/\pi)*(A/N)}\}$. The larger the transmission range is, i.e. the more neighbors, the smaller the number of hops is.

From equations (1) and (2), the energy consumption of one hop with two and (k-1) neighbors of s-bit message are as follows, respectively:

$$(A/N) * \{E_{amp} * s * (\sqrt{3/\pi})^2\} + (E_{elec} * s * 2). \tag{6}$$

$$(A/N) * \{E_{amp} * s * (\sqrt{k/\pi})^2\} + \{E_{elec} * s * (k-1)\}. \tag{7}$$

The number of hops to the same destination, i.e. the same distance *d*, by two and (k-1) neighbors are $\{d/\sqrt{(3/\pi)*(A/N)}\}$ and $\{d/\sqrt{(k/\pi)*(A/N)}\}$, respectively. When the number of hops applies to (6) and (7), the total energy to the destination can be represented as follows:

Total energy with two neighbors, TE_2, is

$$\{E_{amp} * s * d * \sqrt{(3/\pi)*(A/N)}\} +$$
$$\{(E_{elec} * s * 2d) / \sqrt{(3/\pi)*(A/N)}\}. \tag{8}$$

Total energy with (k-1) neighbors, $TE_{(k-1)}$, is

$$\{E_{amp} * s * d * \sqrt{(k/\pi)*(A/N)}\} +$$
$$\{(E_{elec} * s * (k-1)d) / \sqrt{(k/\pi)*(A/N)}\}. \tag{9}$$

Clearly the value of equation (9) is always larger than that of equation (8) when k = 4,…, N if the values of E_{amp}, s, *d*, and E_{elec} are the same in both equations.[4]

Theorem 2 (Other Radio Propagation Models)

The topology with two neighbors consumes the minimum amount of energy to the destination in other radio propagation models.

(Proof)

In different radio propagation environments, only the path loss exponents are different and any known path loss exponents are larger than one [19]. For instance, the path loss exponents are 2 for the "Free Space," 4 to 6 for "Obstructed in building" and 1.6 to 1.8 for "In building line-of-sight." Thus, again, the value of equation (9) is always larger than that of the equation (8) when *k* = 4,…, N.

From both theorems, we can conclude that the minimal energy consumption occurs when the number of neighboring microsensors is two even though it takes a greater number of hops to the destination. However, two neighbor topologies provide a zero-resilient path to the destination, which results in a very high probability of failure to the destination.

5.3 Energy Consumption in the Topology

The total consumed energy depends on the number of generated messages coming from the topology. A possible network topology may take the form of one of three typical topologies (shown in Figure 6): 1) all microsensors are linearly connected; 2) all microsensors are within one transmission range, i.e. fully directly connected; and 3) each microsensor has an average (k-1)-neighbor, i.e. *k* microsensors are within the transmission range r.

If all microsensors adopt the same transmission range and message length, the energy to send and receive a message can be represented by TE_{Tx} and TE_{Rx} respectively. The specific TE_{Tx} and TE_{Rx} value are determined from r, N, A, the desired probability *p*, and the average of non-operational microsensors *q*, the distance to the destination *d*, the length of message s and *α*-connectivity to the destination (see sections 3 and 4). The propagation models and routing algorithms[5] also affect the TE_{Tx} and TE_{Rx} value [19]. The message propagation from source to destination makes a tree, where the source is a root and the destination becomes a leaf. Intermediary microsensors relay messages at most one time and do not retransmit the same message to neighboring microsensors.

Case 1: Linearly connected

The maximum distance for a message to go is (N-1) hops, in a row of N microsensors and (N-1) messages as relay from each microsensor, which result in (N-1) messages. Message propagation spends only one transmission energy in relay to a next microsensor and consumes one receive energy from its one neighbor. The necessary energy is $(N-1) * (TE_{Rx} + TE_{Tx})$.

In two different transmission ranges, say r and r' and r > r', the maximally possible distance is

$$\{(N-1) * r\} - \{(N-1) * r'\} < r'. \tag{10}$$

The inequality of (10), i.e. $\{(N-1)/N\} < (r'/r)$, shows that the difference between two transmission ranges is only (1/N), which is very small. In the first order radio model the difference of consumed energy is

$$(N-1) * \{E_{amp} * s * (r^2 - r'^2)\}. \tag{11}$$

The value of $(r^2 - r'^2)$ is almost zero; thus, the value of (11) becomes zero.

Case 2: Fully directly connected

A source microsensor knows that the destination is one of the neighboring microsensors. To send a message to the destination, a message is transmitted to all neighbors and (N-1) neighboring microsensors hear the message. The total number of messages in this configuration is one transmission and (N-1) microsensors listen to the message. The necessary energy is

$$\{(N-1) * TE_{Rx}\} + TE_{Tx}. \tag{12}$$

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[4] When $k = 3$, the topology is in a row; the difference in consumed energy is almost zero (see 5.3 Energy Consumption in the Topology).

[5] There are many routing algorithms that are based on different assumptions; e.g. one-hop distance clustering based routing, virtual backbone based routing, and etc. But we assume there are no specific routing algorithms, virtual backbones and knowledge like the destination coordinates. Our analysis adopts omni directional propagation as a communication mechanism.

The difference of consumed energy between the two transmission ranges of r and r' is

$$\{E_{amp} * s * (r^2 - r'^2)\}. \qquad (13)$$

Equation (13) is much smaller than equation (11) and becomes zero.

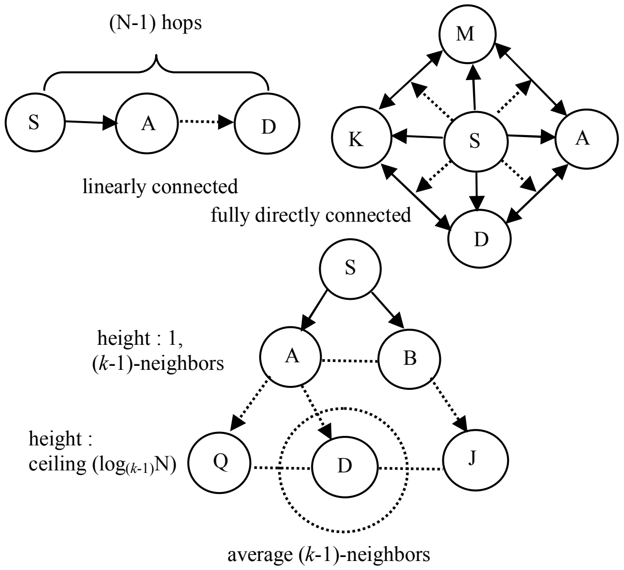

Figure 6. Possible topologies.

Case 3: Average (k-1) neighbors

In this case, the height of a propagation tree is the ceiling ($\log_{(k-1)}N$). Each height except the leaves has $(k\text{-}1)^{height}$ microsensors (e.g. there are $(k\text{-}1)^2$ microsensors at height 2). In the propagation tree, each microsensor plays one of the following roles – a root, a leaf or an intermediary. The number of roots, leaves and intermediaries are as follows:

$$\text{root} = 1$$

$$\text{intermediary} = \sum_{i=0}^{flooring(\log_{(k-1)} N)} (k-1)^i \text{ - root}$$

$$\text{leaf} = \text{N} - \text{intermediary} - \text{root}. \qquad (14)$$

At the leaves, microsensors receive only a message and do not relay an incoming message. A root, i.e. the source node, expends the energy of TE_{Tx}. Each intermediary microsensor consumes $\{(TE_{Rx} * (k\text{-}1)) + TE_{TX}\}$, and receives the same message from (k-1) neighbors and relays it to the neighbors. The necessary energy for this topology[6] is the sum of the energy at the root, the energy at the intermediary, and the energy at the leaves. The consumed energy is as follows:

the energy at the root = TE_{TX}

the energy at the intermediary =

$$\{(k\text{-}1) * TE_{RX} + TE_{TX}\} * \{ \sum_{i=0}^{flooring(\log_{(k-1)} N)} (k-1)^i \text{ - } 1\}$$

the energy at the leaves =

$$\{(k\text{-}1) * TE_{Rx}\} * \{N - \sum_{i=0}^{flooring(\log_{(k-1)} N)} (k-1)^i \}. \qquad (15)$$

The total consumed energy of equation (15) depends on N, (k-1), A, and the ratio of TE_{TX} and TE_{RX}.

Theorem 3 (Energy Consumption by Number of Neighbors)

The topology with a smaller number of neighboring microsensors dissipates less energy than that with more neighbors.

(Proof)

Part A: the second and the third component of equation (15) are *monotonic* increasing functions depending on k, i.e. when k increases, the value of the second and third also increase. Thus, when given N, A, TE_{TX}, and TE_{RX}, the total dissipated energy with smaller neighbors is less than that of more neighbors.

Part B: assume that a directional propagation is possible with the help of some knowledge of the destination and current location. In that case only one microsensor relays the incoming message and remaining (k-1) neighbors do not retransmit, i.e. only dissipate the energy for message reception. This is the same situation as the analysis of energy consumption in the propagation models.

From *part A* and *B*, we can conclude that the topology with the smaller number of neighboring microsensors consumes a less amount of energy than that of more neighbors.

5.4 Energy Consumption by the Connection Probability

Using equations (4) and (5), we can eliminate the uncertainty of connection by limited energy, failures and temporary broken links. To minimize the energy consumption to the destination, we select the transmission range which has the maximum probability in $P[M{=}k]$. In Figure 4, we assume that we need an exact three neighbor topology, in which the transmission range of 0.6 has the highest probability of 0.2. If we need more than three neighbors with higher probability than 90%, the transmission range should be at minimum 0.6 (in Figure 5). However, if we want the topology of three neighbors with more than 95% probability, the transmission range should start from 0.7, not 0.6. The expected total energy consumption is a summation of the two energies: the normal consumed energy under the connection probability and the additional energy to cover deficiency to full connection, i.e. 100% connection probability. If the total consumed energy with 90% probability is y, then the expected total energy is (y+0.11y). In Figure 5, if we need at least three neighbors with 90% probability, the possible transmission ranges are 0.6 to 0.8. We select the minimum transmission of 0.6 for it incurs less energy consumption than that of 0.7 or 0.8, from our analysis in 5.2 and 5.3 in Section 5. When the budget of energy to the destination is given, the minimum transmission range can be decided from equations (4) and (5). If a connection probability is given with other constraints like the energy budget, we can also select the optimal number of neighbors, transmission range, and etc.

(**Example 2**)

We select the same network conditions as those of Figure 3; 400 microsensors which are randomly deployed in the network area of $100m^2$, 10m x 10m. We choose the possible number of neighbors from 2 to 10. The transmission range should be at least 0.89m, i.e. ($\sqrt{(10/12.56)}$ m), from properties 1 and 2, i.e. $\pi r^2(N/A) > 10$, if we assume that an average of two neighboring microsensors is not in operational mode, sleep mode or failure. The transmission range allows the maximum 8-resilient paths to the destination; two are not in operation and eight can be candidates. The ratio between TE_{RX} and TE_{TX} from (1) and (2) can be written as $TE_{TX} = cTE_{RX}$, where $c > 1$. The higher the data rate is, the more energy the

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[6] We assume that the topology is (k-1)-ary full balanced tree.

transmission power requires [16]. The transmission strategy also affects the total energy consumption, i.e. the periodic sleeping dissipates less energy than the non-sleeping strategy [26]. The effects by data transmission rate and sleeping strategy are assumed to be included in the parameters TE_{TX} and TE_{RX}. The data length, s, does not affect the ratio because it is in both equations. We also select the first order radio model and corresponding E_{elec} and E_{amp} values as 50nJ/bit and 100pJ/bit/m^2 [18]. In Figure 7, we change both the network density, i.e. the value of (N/A), and the number of neighboring microsensors, (*k*-1). We monitor the total consumed energy in Figure 8, which depends on the transmission range and the ratio of TE_{TX}/TE_{RX}. Figure 9 shows the total energy in the different network densities. The three figures represent that the more the neighboring microsensors exist, the more energy the network requires; and the higher the ratio of TE_{TX} and TE_{RX} (or the transmission range) is, the more energy the network consumes. With the same number of neighbors, the higher the network density is, the more energy the network consumes.

6. DISCUSSION

In this section, we analyze the required energy when the network partition occurs. When a microsensor in the path to the destination is out of reach, two microsensors (incoming and outgoing neighbors in the path) detect such a disconnection immediately. The topologies of linearly connected or fully connected cases are very simple – a network separation occurs. We discuss the average (*k*-1)-neighbor case below.

6.1 One Intermediary Microsensor is Out of Reach

The microsensor in an outgoing path selects the next candidate path and notifies to a new incoming microsensor. The new incoming microsensor changes its outgoing path. Thus, two transmissions are required: one from an outgoing microsensor and another from a new incoming microsensor as a confirmation. The required energy consumption is $2 * \{(k\text{-}1) * TE_{Rx} + TE_{Tx}\}$.

6.2 The Destination is Disconnected

The incoming microsensor of the destination broadcasts a message to search the destination. A question arises: how much the search has to expand. The search scope depends on the given conditions such as distance (e.g., only one hop or the entire network) or energy amount. The required energy for the search is the same as that described in Section 5 – the same destination and the new source. If the destination is not found, an error message is generated to the source. When search results contain new detour paths to the destination, a lower energy path is selected. If the destination is still within one hop distance, then the energy to find a new path to the destination is $\{((k\text{-}1) * TE_{Rx} + TE_{Tx}) + (k\text{ -}1) * ((k\text{-}1) * TE_{Rx} + TE_{Tx})\}$, i.e. $\{k * ((k\text{-}1) * TE_{Rx} + TE_{Tx})\}$. All neighbors, (*k*-1), transmit the search message and receive the same requests from all its neighbors, (*k*-1).

7. CONCLUSION

We show that when it has fewer neighbors, microsensor topology dissipates smaller energy even though it has to experience more hops to the destination. Thus, an energy efficient topology must have fewer neighbors. As well, the connection probability incurs additional energy to the destination. Using the connection probability, we can decide on an energy efficient topology using the following parameters: the number of neighbors, transmission range, network density, the maximum allowed energy and etc. We plan to analyze the energy consumption induced by microsensor mobility in future work.

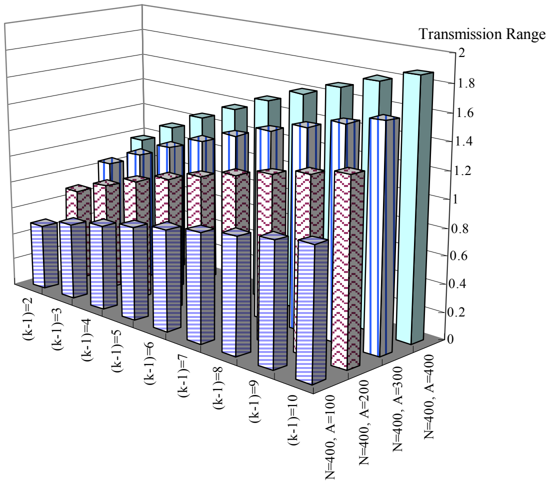

Figure 7. Total energy by transmission range.

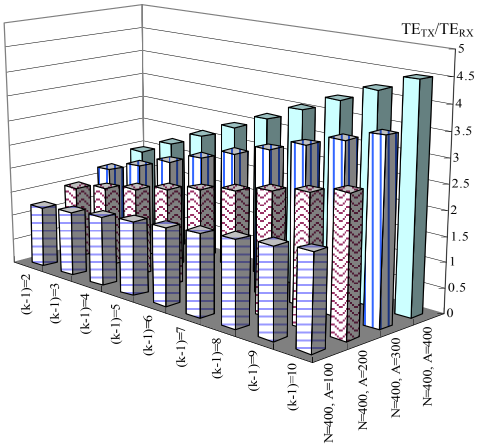

Figure 8. Total energy by TE_{TX}/TE_{RX}.

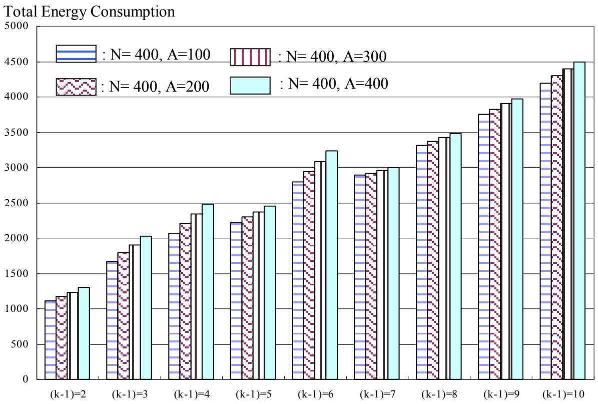

Figure 9. Total energy by different density, 1 to 4.

8. REFERENCES

[1] Aron, Ionut D. and Gupta, S. K. S. On the Scalability of on-demand routing protocols for mobile ad hoc networks: Analytical Study. *Journal of Interconnection Networks*. 2(1): 5-30, March 2001.

[2] Bluetooth Project. http://www.bluetooth.com. 1999.

[3] Banerjee, S. and Misra, A. Minimum Energy Paths for Reliable Communication in Multi-hop Wireless Networks. In *Proceedings of ACM Mobihoc*, Computer Science, SITE, University of Ottawa, June 1999.

[4] Bhardwaj, M., Garnett, T. and Chandrakasan, A. Upper Bounds on the Lifetime of Sensor Networks. *IEEE International Conference on Communications*, pages 785 - 790, 2001.

[5] Chandrakasan, A., Amirtharajah, R., Cho, S. H., Goodman, J., Konduri, G., Kulik, J., Rabiner, W. and Wang, A. Design Considerations for Distributed Microsensor Systems. *IEEE 1999 Custom Integrated Circuits Conference (CICC)*, pages 279–286, May 1999.

[6] Chandrakasan, A., Min, R., Bhardwaj, M., Cho, S. H. and Wang, A. *Power Aware Wireless Microsensor Systems*. Keynote Paper, ESSCIRC, 2002.

[7] Chang, J. H. and Tassiulas, L. Energy conserving routing in wireless ad-hoc networks. In *Proceedings of IEEE INFOCOM 2000*, Tel Aviv, Israel, pages 22-31, March 2000.

[8] Chen, B., Jamieson, K., Balakrishnan, H. and Morris, R. Span: an energy-efficient coordination algorithm for topology maintenance in ad hoc wireless networks. *MobiCom 2001*, Rome, Italy, pages 70-84, July 2001.

[9] Clare, L., Pottie, G. and Agre, J. R. Self-Organizing Distributed Sensor Networks. *SPIE Conference on Unattended Ground Sensor Technologies and Applications*, pages 229–237, April 1999.

[10] Clark, B. N., Colbourn, C. J. and Johnson, D. S. Unit Disk Graphs. *Discrete Mathematics*, 86:165–177, 1990.

[11] Estrin, D. http://nesl.ee.ucla.edu/tutorials/mobicom02.

[12] Estrin, D. and Govindan, R. Next century challenges: scalable coordination in sensor networks. *MobiCom 1999*, Seattle, WA, pages 263-270, August 1999.

[13] Kahn, J. M., Katz, R. H. and Pister, K. S. J. Next Century Challenges: Mobile Networking for Smart Dust. In *Proc. Mobicom 1999*, pages 483–492, 1999.

[14] Maltz, D., Broch, J., Jetcheva, J. and Johnson, D. The Effects of On-Demand Behavior in Routing Protocols for Multi-HopWireless Ad Hoc Networks. *IEEE JSAC*, 17(8): 1439–1453, August 1999.

[15] Perkins, C. and Royer, E. Ad-hoc On-Demand Distance Vector Routing. *Proc. 2nd IEEE Workshop Mobile Comp. Sys. and Apps.*, pages 90–100, Feb. 1999.

[16] Proakis, J. G. *Digital Communications*. New York, NY, McGraw-Hill, 1995.

[17] Rabaey, J., Ammer, J., da Silva, J. L. and Patel, D. PicoRadio: Ad-hoc wireless networking of ubiquitous low-energy sensor/monitor nodes. *IEEE Computer Society Workshop on VLSI 2000*, Orlando, FL, pages 9-12, April 2000.

[18] Rabiner, W., Chandrakasan, A. and Balakrishnan, H. Energy-Efficient Communication Protocol for Wireless Microsensor Networks. *Proc. IEEE Hawaii Int. Conf. on System Sciences*, January 2002.

[19] Rappaport, T. S. *Wireless Communications: Principles and Practice*. Englewood Cliffs, NJ: Prentice-Hall, 1996.

[20] Savvides, A., Han, C. C. and Srivastava, M. Dynamic finegrained localization in ad-hoc networks of sensors. *MobiCom 2001*, Rome, Italy, pages 166 – 179, July 2001.

[21] Schurgers, C., Tsiatsis, V., Ganeriwal, S. and Srivastava, M. Topology management for sensor networks: Exploiting latency and density. In *Proceedings of The Third ACM International Symposium on Mobile Ad Hoc Networking and Computing (MobiHoc)*, 2002.

[22] Wieselthier, J., Nguyen, G. D. and Ephremides, A. On the Construction of Energy-Efficient Broadcast and Multicast Trees in Wireless Networks. In *Proceedings of IEEE INFOCOM2000*, pages 146–156, June 2002.

[23] Williams, R. *The Geometric Foundation of Natural Structure: A Source Book of Design*. Dover Pub. Inc., New York, pages 51–52, 1979.

[24] Wireless Integrated Network Sensors, University of California, Los Angeles. Available: http://www.janet.ucla.edu/WINS

[25] Xu, Y., Heidemann, J. and Estrin, D. Geography-informed energy conservation for ad hoc routing. *MobiCom 2001*, Rome, Italy, pages 70-84, July 2001.

[26] Zhu, Jin and Papavassiliou, S. On the Connectivity Modeling and the Tradeoffs between Reliability and Energy Efficiency in Large Scale Wireless Sensor Networks. *Proc. IEEE Wireless Communications and Networking(WCNC)*, pages 1260–1265, March 2003.