Archiv für Elektrotechnik, Vol. 75, 1992, pp. 255-260

Time Domain Simulation of Skin- and Proximity-Effect in Multiconductor Transmission Lines

by

E. Grotelüschen, K.-P. Dyck, H. Grabinski

Abstract - **A new method of modeling the skin- and proximity-effect within transmission lines is presented. This technique is used in connection with an algorithm for a fast time domain simulation of the signal propagation in lossy interconnect systems. The frequency-dependence of the line parameters is taken automatically into consideration by the chosen approach and does not need to be known in advance. In addition the simulation of line systems in a nonlinear circuit environment (such as digital circuits) is possible, since the algorithm operates in the time domain. The validity of the computed results in the time domain has been proven by a comparison with results gained from frequency domain simulations. Simulation results are given for different planar transmission line systems. It was found that the influence of skin- and proximity-effect is negligible for lines currently used in VLSI circuits. This is also true in cases where the dimensions of the cross-section of the lines are significantly larger then the depth of penetration caused by the skin-effect (e.g. on PCBs).**

I. Introduction

Transmission lines with frequency-dependent losses caused by skin-effect have been investigated for many years. Most of the published approaches operate in the frequency domain [1 - 4]. But since nearly all line systems are embedded into a *nonlinear* circuit environment (especially in case of digital circuits), the use of *linear* integral transformations, such as Fourier transformation, are not practical. Therefore only time domain simulation techniques are further considered.

In 1976 Miersch and Ruehli proposed a transmission line model consisting of lumped elements to simulate the influence of frequency-dependent losses of single lines [5]. In later publications by the same authors capacitances and inductances were substituted by lossless transmission lines to avoid time consumptive and inaccurate numerical integrations [6, 7].

Another approach done by Yen et al. was to divide the whole cross section of a single transmission line into subsections, from which equivalent circuits consisting of resistors and inductors were derived [8]. The lossy transmission line is then described by a lossless transmission line, representing the line delay, and the above mentioned equivalent circuits to represent the influences of the skin-effect. Using these equivalent circuits in conjunction with the lossless transmission lines, the transient behaviour of the line can be simulated in the time domain.

Djordjević et al. presented a convolution technique using Green's functions for lines, by which lossy transmission line systems can be calculated [9]. Applying this method it is possible to take the influences of skin- and proximity-effect into account, provided that the frequency-dependence of the line parameters is known in advance.

Recently some simulation techniques were presented using the "Finite element method" to compute appropriate equivalent circuits, which were then used within general purpose network analysis programs like SPICE to represent the lines [10, 11].

In particular three disadvantages arise when using the above mentioned simulation methods. First, most interconnections consist of coupled line systems. Thus, analyzing *single* transmission lines only is not sufficient. Merely the method proposed by Djordjević et al. [9] provides the simulation of lossy line *systems*. Second, using the methods described above, the frequency-dependence of line parameters have to be calculated in advance. In general, this can be very difficult and time consuming (except for cylindrical conductors of simple geometries such as coaxial cables). Third, describing the frequency-dependence of line parameters by equivalent circuits is not useful at all, since the calculation of networks containing lumped capacitors and inductances requires the use of numerical integration routines. For large line systems methods of numerical integration in general fail due to excessive computer runtime and the occurence of numerical instabilities. The main reason for this is, that a network is described by a quasistationary approach using a system of *ordinary* differential equations, whereas a transmission line system is described by a set of *partial* differential equations. Thus, within a network all events take place simultaneously (in contrast to lines). Therefore the attempt, to approximate lines or line systems by network elements will lead to the previously mentioned problems. Note, that the disadvantages of numerical integration routines also arise with the method proposed by Djordjević et al. [9]: due to the convolution technique lots of numerical integrations are necessary.

In order to avoid the problems mentioned above, this paper presents a new time domain simulation technique for lossy line systems, which includes the consideration of skin- and proximity-effect. The basic idea of this new approach is to model the original line system (i.e. the line system with *frequency-dependent* line parameters) by an extended line system with *constant* line parameters. The extended line system can then be simulated by an algorithm which takes into account the non-quasistationary behaviour of coupled line systems [14]. A brief description of this algorithm will be given in chapter II.

In chapter III a transmission line model is introduced which provides the simulation of skin- and proximity-effect and which can be simulated by means of the algorithm described in chapter II. Some simulation results are presented in chapter IV.

II. The Algorithm

Due to its importance for this work, a very efficient algorithm for time domain simulation of lossy transmission lines with constant line parameters will be described briefly. For a more detailed information cf. [14].

It can be shown, that for most interconnections, especially on chips, one can assume that the quasistationary case exists within the cross section of a system of parallel uniform lines [13]. Thus, we can restrict ourselves to cases where the quasi-TEM approximation is valid. A system of uniform lossy transmission lines in a nonhomogeneous medium with respect to the cross section is then described by a system of partial differential equations:

$$\frac{-\partial \boldsymbol{V}}{\partial x} = \boldsymbol{R}'\boldsymbol{I} + \boldsymbol{L}'\frac{\partial \boldsymbol{I}}{\partial t} \ , \tag{1a}$$

$$\frac{-\partial \boldsymbol{I}}{\partial x} = \boldsymbol{G}'\boldsymbol{V} + \boldsymbol{C}'\frac{\partial \boldsymbol{V}}{\partial t} \ . \tag{1b}$$

$\boldsymbol{V}$ and $\boldsymbol{I}$ are the column matrices of line potentials and line currents. $\boldsymbol{R}'$, $\boldsymbol{L}'$, $\boldsymbol{G}'$ and $\boldsymbol{C}'$ are square matrices of the resistance coefficients, inductance coefficients, conductance coefficients, and Maxwell`s capacitance coefficients, all per unit length, respectively. In general $\boldsymbol{R}'$ is a full matrix. Only for the case of an ideal ground plane is $\boldsymbol{R}'$ a diagonal matrix containing the ohmic resistances per unit length of the lines. As the conductance matrix $\boldsymbol{G}'$ is not of importance for all further considerations, it is assumed to be zero for the following.

Since an analytical solution of (1) is not known, the idea of the algorithm is to divide the lossy line system into short segments in the direction of the wave propagation. Each segment then consists of a lossless line system and an attenuation network (Fig. 1).

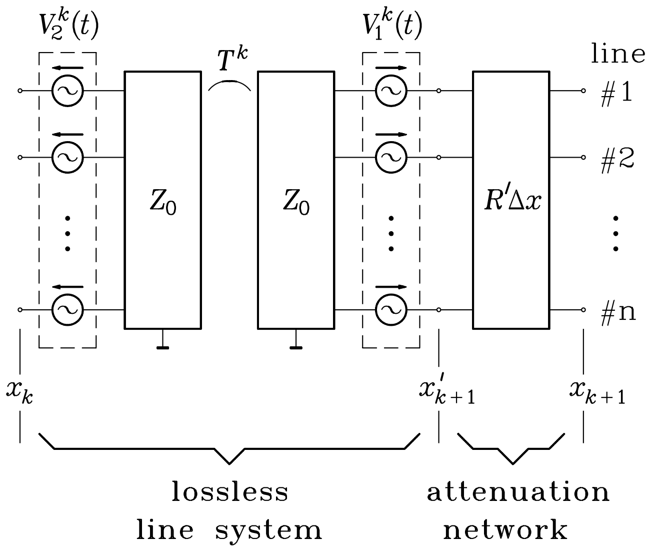

Fig. 1. Circuit model of the *k*-th segment of a lossy line system.

The system of coupled differential equations of the *lossless* line system can be transformed into a set of decoupled equations by an appropriate linear transformation (similarity transformation). For the lossless system a solution can then be found in accordance with the "Solution of d'Alembert" which is well-known for a lossless single line. This analytical solution for the line system leads to the circuit depicted in Fig. 1.

Let x_k be the start position and $x_{k+1}^{/}$ be the end position of the lossless line system of the *k*-th segment, then voltages $\boldsymbol{V}_{\mathbf{1}}^{\boldsymbol{k}}(t)$ and $\boldsymbol{V}_{\mathbf{2}}^{\boldsymbol{k}}(t)$ can be expressed by voltages and currents at the boundaries of the line system (Note: *k* is not an exponent but an index !):

$$\boldsymbol{V}_1^k(t) = \boldsymbol{V}\left(x_k\,,\,\boldsymbol{1}t - \boldsymbol{T}^k\right) + \boldsymbol{Z}_0 \otimes \boldsymbol{I}\left(x_k\,,\,\boldsymbol{1}t - \boldsymbol{T}^k\right)\,, \tag{2a}$$

$$\boldsymbol{V}_2^k(t) = \boldsymbol{V}\left(x_{k+1}^{/}\,,\,\boldsymbol{1}t - \boldsymbol{T}^k\right) - \boldsymbol{Z}_0 \otimes \boldsymbol{I}\left(x_{k+1}^{/}\,,\,\boldsymbol{1}t - \boldsymbol{T}^k\right)\,. \tag{2b}$$

Here $\boldsymbol{1}$ is the identity matrix and $\boldsymbol{Z}_0$ is the matrix of characteristic impedances. The matrix $\boldsymbol{T}^k$ represents the delay times of the line system of the *k*-th segment. The linear operator $\otimes$ acts as a simple matrix multiplication in the case of $\boldsymbol{T}^k \equiv \boldsymbol{0}$. Else $\otimes$ extends $\boldsymbol{I}$ to a square matrix, performs the matrix multiplication with $\boldsymbol{Z}_0$, takes the diagonal elements only and puts them into a column matrix again [14]. The description of a lossless line system (eq. 2) is then in completely analogy to that of a single line.

For simplification let all segments of the entire line system have the same length Δx. Considering the attenuation network shown in Fig. 1, voltages $\boldsymbol{V}_1^k(t)$ and $\boldsymbol{V}_2^k(t)$ of the *k*-th segment can be expressed by appropriate voltages of the *k*-th, *(k-1)*-th and *(k+1)*-th segment at times $\boldsymbol{1}t - \boldsymbol{T}$:

$$\boldsymbol{V}_1^k(t) = \left(\boldsymbol{1} - \boldsymbol{A}_1\right) \otimes \boldsymbol{V}_2^k\left(\boldsymbol{1}t - \boldsymbol{T}\right) + \boldsymbol{A}_2 \otimes \boldsymbol{V}_1^{k-1}\left(\boldsymbol{1}t - \boldsymbol{T}\right) \;, \tag{3a}$$

$$\boldsymbol{V}_2^k(t) = \left(\boldsymbol{1} - \boldsymbol{A}_1\right) \otimes \boldsymbol{V}_1^k\left(\boldsymbol{1}t - \boldsymbol{T}\right) + \boldsymbol{A}_2 \otimes \boldsymbol{V}_2^{k+1}\left(\boldsymbol{1}t - \boldsymbol{T}\right) \;, \tag{3b}$$

where $\boldsymbol{A}_1$ and $\boldsymbol{A}_2$ are defined to be:

$$\boldsymbol{A}_1 := \boldsymbol{A}_2 := \boldsymbol{Z}_0 \boldsymbol{Z}^{-1} \tag{4}$$

with
$$\boldsymbol{Z} := \boldsymbol{Z}_0 + \frac{1}{2}\Delta x \boldsymbol{R}' \;. \tag{5}$$

The matrices $\boldsymbol{A}_1$ and $\boldsymbol{A}_2$ describe the attenuation of the lossy line system. ($\boldsymbol{A}_1$ and $\boldsymbol{A}_2$ are not equal for the case $\boldsymbol{G}' \neq \boldsymbol{0}$). Using equations (2) - (5) the line system can now be simulated in the time domain. The required number of segments essentially depends on the magnitude of the ohmic line losses.

The benefit of the described algorithm is that *no numerical integrations at all* are required for the simulation of line systems. As a result, the computer runtime becomes extremely short and no numerical instabilities arise. In addition, the accuracy of the results is very high. The algorithm is implemented in a computer program called LISIM. LISIM has the capability to simulate coupled lossy line systems of arbitrary size in a nonlinear circuit environment [15, 16].

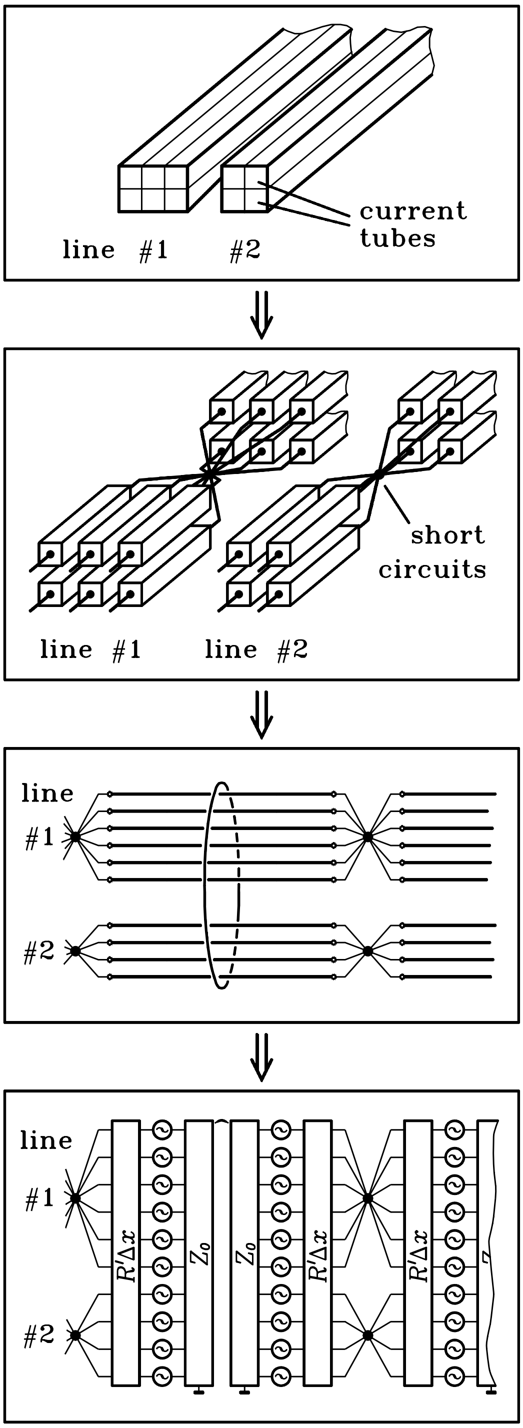

Fig. 2. A 2-line system modeled by a 10-line system of coupled current tubes.

III. A Model for Simulation of Skin- and Proximity-Effect

To simulate the transient behaviour of a line system taking into account skin- and proximity-effect, each conductor is now divided into a number of parallel lines, called current tubes. Thus, each line itself (of a line system) is modeled by a line *system* of coupled current tubes (Fig. 2). The current density in each tube is assumed to be approximately constant with respect to its cross section. This is guaranteed if the geometrical dimensions of each current tube is chosen to be smaller than the skin depth.

For a sufficiently high conductivity of the line material, (dielectric) displacement currents in the conductor can be neglected as compared to conduction current. Thus, coupling of current tubes is essentially galvanic and magnetic. In the model the galvanic coupling is achieved by a short-circuit of all current tubes belonging to the same line in each segment, cf. Fig. 2. Thus, the current tube voltages at a position x are the same for all current tubes of a line. It is evident, that the whole current of the line equals the sum of all tube currents. The short-circuit mentioned above can be realized by introducing a new Matrix $\boldsymbol{E}$ as follows:

Let the original line system be composed of n lines and let the i-th line be divided into p_i current tubes ($p_i \geq 1$). A non-ideal ground plane may be divided into a reference line and p_0 current tubes (cf. [3]). This results in an extended system with $q = \sum_{i=0}^{n} p_i$ current tubes. A new matrix $\boldsymbol{E}$ with the dimension ($q \times n$) is then introduced, whose elements are defined as:

$$E_{i,j} = \begin{cases} 1, & \text{if current tube } i \text{ belongs to line } j, \\ 0, & \text{else.} \end{cases} \tag{6}$$

With $\boldsymbol{E}^T$ being the transpose of matrix $\boldsymbol{E}$, equation (4) now read:

$$\boldsymbol{A}_1 = 2\boldsymbol{Z}_0\boldsymbol{Z}^{-1}\left[\boldsymbol{1} - \boldsymbol{E}\left(2\boldsymbol{E}^T\boldsymbol{Z}^{-1}\boldsymbol{E}\right)^{-1}\boldsymbol{E}^T\boldsymbol{Z}^{-1}\right] \quad , \tag{7a}$$

$$\boldsymbol{A}_2 = \boldsymbol{Z}_0\boldsymbol{Z}^{-1}\boldsymbol{E}\left(\boldsymbol{E}^T\boldsymbol{Z}^{-1}\boldsymbol{E}\right)^{-1}\boldsymbol{E}^T\boldsymbol{Z}^{-1} \quad . \tag{7b}$$

Herein $\boldsymbol{Z}_0$ and $\boldsymbol{Z}$ have the dimension ($q \times q$). If the lines are not divided into current tubes (i.e. $q = n$), the matrix $\boldsymbol{E}$ equals the identity matrix and equations (7) and (4) are identical.

As an example Fig. 2 shows a 2-line system ($n = 2$): lines #1 and #2 are divided into $p_1 = 6$ and $p_2 = 4$ current tubes, respectively. This results in $q = 10$. Matrix $\boldsymbol{E}^T$ is then given by:

$$\boldsymbol{E}^T = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix} \quad .$$

To determine the inductance coefficients of the extended line system (i.e. q-line system, $q \geq n$), the stationary values of the self and mutual inductances per unit length of each tube have to be calculated. In case of coaxial and rectangular conductors this can be done by using analytical formulas [3].

Even though coupling capacitances between two current tubes in the same line can be neglected due to the galvanic coupling mentioned above, the algorithm nevertheless requires *all* capacitance coefficients of the q-line system. Therefore the capacitance between any two lines has been distributed over all current tubes belonging to those lines. Due to the short-circuits mentioned above this distribution of the capacitances can be varied over a wide range without changing the simulation results. But due to numerical reasons this capacitance should be divided into physically reasonable partial capacitances, e.g. each partial capacitance has the same magnitude.

IV. Some simulation results

The performance of the method described above will be demonstrated through examples of signal propagation taking skin- and proximity-effect into account. In the given examples all peripheral elements (except signal sources and ohmic load resistances) are omitted, so that only effects resulting from the influences of lines are observed.

To prove the validity of the computed results in the first example the output signal of a coaxial cable was simulated. Since the frequency dependence of the line parameters of a coaxial cable is well-known and can be determined by the analytical formulas (8), the pulse response can also be computed in a simple way in the frequency domain using 'Fast Fourier Transformation' (FFT). The complex impedance per unit length of the center conductor is given by:

$$R^{/} + j\omega L_i^{/} = R_0^{/} \frac{k r_i}{2} \frac{J_0(k r_i)}{J_1(k r_i)} \qquad \text{with} \qquad k = (1-j)\sqrt{\frac{\omega\sigma\mu}{2}} = \frac{1-j}{\delta} \tag{8}$$

Herein $R_0^{/}$ is the dc resistance per unit length and r_i the radius of the center conductor. J_0 and J_1 are complex Bessel functions of the first kind and δ is the skin depth. The coaxial cable is assumed to be 600 m long. Its center conductor has a radius of 0.45 mm and is made up of copper (σ_{CU} = $56 \cdot 10^6$ S/m). The outer conductor has an inner radius of 1.475 mm and was assumed to have infinite conductivity. The dielectric is assumed to be lossless with a relative dielectric constant of 2.15. This leads to a nominal characteristic resistance of about 50 Ω. The cable is stimulated by an ideal signal source and loaded on the output with a resistance of 50 Ω. In this example the input voltage is a pulse with a rise and fall time of 25 ns and a high time of 1 µs.

For the time domain simulation the cross section of the center conductor was divided into 3 and 10 concentric rings, respectively. As a result of the skin-effect the variation of the current density is much stronger in the outer rings than in the center rings. Therefore the thickness of the outer rings should be chosen to be smaller then the radius of the center rings. In this example all the concentric rings were chosen so that the resistances (per unit length) of all the current tubes were the same.

Results of the simulations are shown in Fig. 3. Curves #1 and #2 show the signal response of the time domain simulation using q = 3 and q = 10 current tubes, respectively. Curve #3 shows the response of the frequency domain simulation. In addition, curve #4 shows the calculated response without

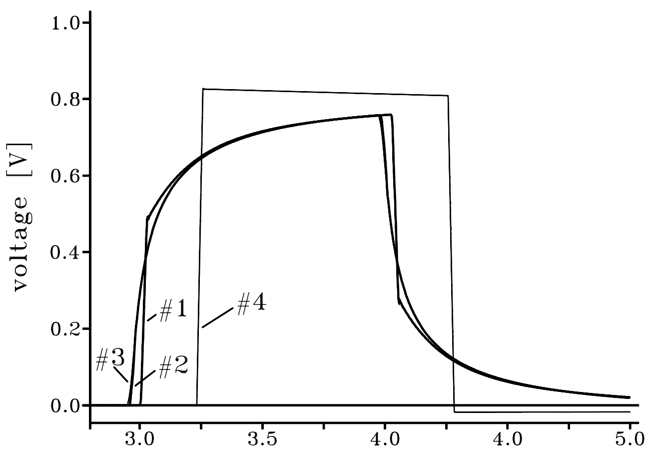

Fig. 3. Output voltages of a coaxial cable. Curves #1 and #2 represent the time domain simulation result; curves #3 and #4 show the results of the frequency domain simulation.

considering any skin-effect (i.e. only dc values of line parameters were used). For all simulations the coaxial line was segmented into 100 segments.

Due to the small mismatch at the output, the voltage at the end of the pulse is not exactly equal to zero but negative for the case of the frequency independent computation (curve #4). Excellent agreement has been obtained between the time domain and the frequency domain simulation results for the case of q = 10 (curve #2 and #3). As a result of skin-effect the (inner) inductance of the cable decreases and the speed of propagation increases with frequency. Therefore the delay time is about 270 ns shorter than if skin-effect is absent. Beyond this, the effective resistance of the center conductor increases due to current displacement. This leads to dispersion effects which can be recognized from the shape of the computed curves. Even for the case of q = 3 current tubes (curve #1) the computed output voltage matches the correct result very closely.

In the second example a line system consisting of two coupled copper strip-type transmission lines at a temperature of 77 K is analyzed. A similar example was proposed and (in case of a single line) simulated by Goshal and Smith [11]. As a result of the high conductivity of copper at 77 K (σ_{CU} ≈ 450 S/µm), the skin depth is very small compared to the geometrical dimensions of the line. Therefore this example is well-suited to show the influence of skin- and proximity-effect in multiconductor transmission lines.

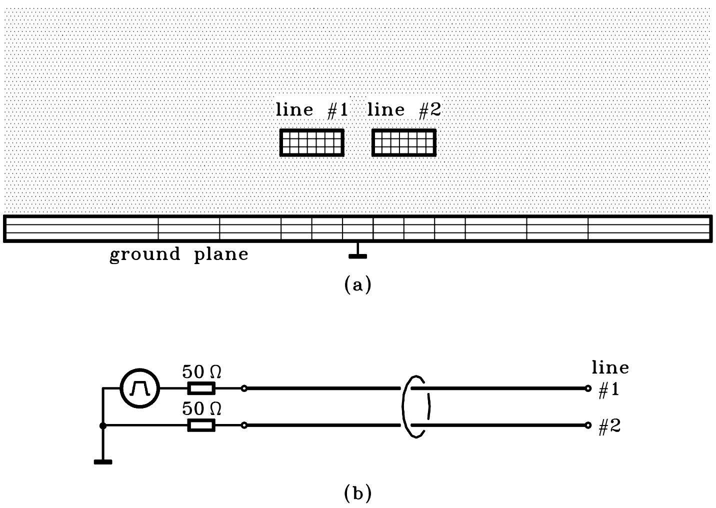

Fig. 4. A line system consisting of two coupled lines.

Both lines of the analyzed system have a width of 5 µm, a thickness of 2 µm and a length of 10 cm. The spacing between the lines is

2.5 µm. The entire line system is embedded in a homogeneous polyimide dielectric with a relative dielectric constant of 3.5 (Fig. 4a). In this example the lines are driven by a source impedance of 50 Ω. The far ends of the transmission lines are left open (Fig. 4b). The stimulating source voltage for line #1 is a pulse with a rise and fall time of 0.1 ns and a high time of 0.9 ns. The source voltages of line #2 is chosen to be constant zero for all times.

For the time domain simulation each conductor is uniformly divided into 21 current tubes with rectangular cross sections. The ground plane is divided into a reference line and p_0 = 38 current tubes. This results in q = 80. The number of segments is chosen to be 100 for the entire line system. The computed signal response is shown in Fig. 5. Curves #1 and #2 show the computed output voltages of lines #1 and #2, respectively. To show the influences of skin- and proximity-effect the same simulation was done without dividing the conductors into current tubes. The results of the latter simulation are depicted in Fig. 5, curves #3 and #4.

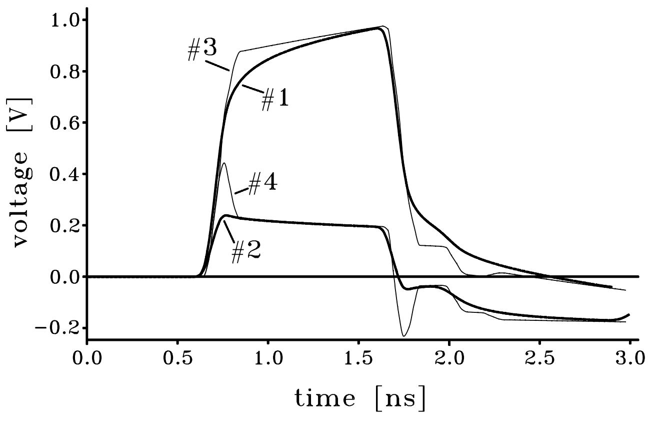

Fig. 5. Output voltages of a 2-line system at 77 K. Curves #1 and #2 show the signal responses if skin- and proximity-effect are considered; curves #3 and #4 show the results if not.

Due to capacitive coupling, a relatively high voltage is observed on the coupled line #2. This voltage remains almost constant during the pulse. In addition, in the case when skin- and proximity-effect are not considered, there are significant voltage peakes on this line at the signal slopes. As a result of dispersion these peakes are leveled down (due to skin-effect). The delay time of the signals is 0.62 ns if the frequency dependence is considered and 0.65 ns if not. In contrast to the first example, the reduction in delay time due to skin- and proximity-effect is only of little importance. Furthermore it can be shown that the influence of the non-ideal ground plane on the coupling is small. The computation of this example took 2580 s CPU-time on a computer operating with 12 Mflops.

The third example is a line system which may be used in current VLSI circuits. It consists of nine coupled lines. All lines have a width of 5 µm, a thickness of 3000 Å and a length of 1 cm. The distance from line to line (center to center) is 8 µm. The conductors are assumed to be of aluminium ($\sigma_{Al} \approx 35$ S/µm). All required data (capacitances, inductances etc.) were obtained from computations for the "slow-wave" mode [17][18]. All lines except the center line (the 5th line) were stimulated with identical signals (simultaneous switching) while the 5th line (center line) was grounded at the input. The rise time of the input voltage was 0.1 ns, the high time 4.4 ns and the fall time 0.5 ns. The line system was divided into 50 segments and each line was divided into 5 current tubes (thus: q = 45).

For simplification purposes only the computed output voltages of lines #4 (curves #1 and #3) and #5 (curves #2 and #4) are shown in Fig. 6. Curves #1 and #2 represent the results if skin- and proximity-effect are considered, and curves #3 and #4 show the output voltages for frequency independent line parameters.

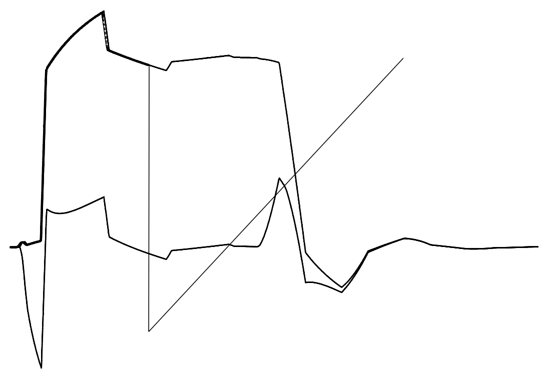

Fig. 6. Output voltages of a 9-line system. Curves #1 and #2 represent the computed response if skin- and proximity-effect are considered; curves #3 and #4 show the results if not.

In this example the skin depth is relatively large compared to the geometrical dimensions (cross section) of the conductors ($\delta \approx 2.7$ µm for f = 1 GHz). Therefore the current density is almost constant with respect to the cross section and the influence of skin- and proximity-effect is very small. As a result the curves are hardly distinguishable from one another. The delay time is about 140 ps and 190 ps, respectively. However, due to magnetic coupling a significant voltage rise can be observed only after 580 ps. Reflexions are observable after 3 × 580 ps = 1.74 ns. The increasing influence of the coupling on the propagation of fast signals can be observed especially at the signal slopes.

As expected, the influence of skin- and proximity-effect is negligible. If the dimensions of the conductor cross section are chosen to be larger than in the last example (e.g. on PCBs) the current density in the conductors is no longer constant (with respect to the cross section) and the effective ohmic resistance increases with frequency. But due to the small dc resistance of the conductors, the line system is almost lossless and skin- and proximity-effect have no influence on the output signal shape. Observable is that only the delay time reduces fractionally due to the decrease of the inner inductance.

V. Conclusions

This paper has presented an algorithm for the time domain simulations of wave propagation along coupled lossy line systems taking the influences of skin- and proximity effect into account. For the algorithm the entire line system is partitioned into segments, each subdivided into a lossy part, consisting of an ohmic network, and a lossless part, described by a lossless line system. In addition, to take skin- and proximity-effect into account, each conductor (including the ground plane) is further divided into current tubes, which are essentially coupled magnetically and galvanically. The frequency-dependence of the line parameters is automatically taken into consideration with this approach. Since the algorithm exclusively operates in the time domain, the influence of a nonlinear circuit environment (such as in digital circuits) can be easily taken into account.

Three examples with their respective simulation results were presented in this paper. In the first example the accuracy of the computed results was proven by a comparison with an appropriate simulation in the frequency domain (using FFT) for the case of a coaxial cable. Two further examples demonstrated the influence of skin- and proximity-effect on the transient behaviour of planar line systems. In case of commonly used VLSI interconnect systems it was found that this influence was negligible, because the geometrical dimensions of the conductor cross sections are in general smaller than the skin depth. With increasing geometrical dimensions (e.g. line systems on boards) the current density in the conductors is no longer constant within the cross section, but since such line systems produce small losses only, skin- and proximity-effect have nearly no influence on the output signal shape.

The influence of skin-effect in the Si-substrate of large VLSI circuits [17, 18] has not been explicitely examined in this paper. In this case frequency-dependent shunt conductances must also be taken into account. This will be subject of further investigations.

Archiv für Elektrotechnik, Vol. 75, 1992, pp. 255-260

References

[1] R. L. Wigington and N. S. Nahman, "Transient analysis of coaxial cables considering skin effect," *Proc. IRE*, vol. 45, pp. 166-174, Feb. 1957.

[2] N. S. Nahman and D. R. Holt, "Transient analysis of coaxial cables using the skin effect approximation $A+B\sqrt{s}$," *IEEE. Trans. Circuit Theory*, vol. CT-19, pp. 443-451, Sept. 1972.

[3] W. T. Weeks, L. L. Wu, M. F. McAllister and A. Singh, "Resistive and inductive skin effect in rectangular conductors," *IBM J. Res. Develop.*, vol. 23, pp. 652-660, Nov. 1979.

[4] J. Chilo, G. Angenieux, C. Monllor, "Proximity effects of interconnection lines in high speed integrated logic circuits," *Proc. 13th European Microwave Conference*, Nürnberg, September 5-8, 1983, pp. 369-372.

[5] E. F. Miersch and A. E. Ruehli, "Analysis of lossy coupled transmission lines," *IBM Tech. Discl. Bull.*, vol. 19, pp. 2363-2365, Nov. 1976.

[6] A. J. Gruodis, C. W. Ho, E. F. Miersch and A. E. Ruehli, "Delay line approach for analyzing lossy transmission lines," *IBM Tech. Discl. Bull.*, vol. 19, pp. 2366-2368, Nov. 1976.

[7] P. A. Brennan and A. E. Ruehli, "Time-domain skin-effect model using resistors and lossless transmission lines," *IBM Tech. Discl. Bull.*, vol. 21, pp. 2162-2163, Oct. 1978.

[8] C.-S. Yen, Z. Fazarinc and R. L. Wheeler, "Time-domain skin-effect model for transient analysis of lossy transmission lines," *Proc. IEEE*, vol. 70, pp. 750-757, July 1982.

[9] A. R. Djordjević, T. K. Sarkar and R. F. Harrington, "Analysis of lossy transmission lines with arbitrary nonlinear terminal networks," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-34, pp. 660-666, June 1986.

[10] G. I. Costache, "Finite element method applied to skin-effect problems in strip transmission lines," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-33, pp. 1009-1013, Nov. 1987.

[11] U. S. Goshal and L. N. Smith, "Skin effects in narrow copper microstrip at 77 K," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-36, pp. 1788-1795, Dec. 1988.

[12] P. Waldow and I. Wolf, "The skin-effect at high frequencies," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-33, pp. 1076-1082, Oct. 1985.

[13] G. L. Matthaei, K. Kiziloğlu, N. Dagli and S. I. Long, "The nature of the charges, currents, and fields in and about conductors having cross-sectional dimensions of the order of a skin depth," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-38, pp. 1031-1035, Aug. 1990.

[14] H. Grabinski, "An algorithm for computing the signal propagation on lossy VLSI interconnect systems in the time domain," *INTEGRATION, the VLSI journal*, vol. 7, pp. 35-48, April 1989.

[15] K.-P. Dyck and H. Grabinski, "LISIM - A simulator for time domain simulation of lossy transmission line systems in a nonlinear circuit environment," *IEEE CompEuro*, Hamburg, May 8-12, 1989, pp. 5.82-5.85.

[16] K.-P. Dyck and H. Grabinski, "A Time domain simulation technique for lossy transmission line systems in VLSI circuit Simulation," *ESM '89*, Rome, June 7-9, 1989, pp. 317-323.

[17] H. Hasegawa, M. Furukawa and H. Yanai, "Properties of microstrip line on Si-SiO_2 system," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-19, pp. 869-881, Nov. 1971.

[18] H. Hasegawa and S. Seki, "Analysis of interconnection delay on very high-speed LSI/VLSI chips using MIS microstrip line model," *IEEE Trans. Electron. Devices*, vol. ED-31, pp. 1954-1960, 1984.